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Biophysics. — *The effect of radiation on light emission by luminous bacteria.* **II.** By A. J. KLUYVER, G. J. M. VAN DER KERK and A. VAN DER BURG.

(Communicated at the meeting of September 26, 1942.)

5. *The photochemical action in the ultraviolet region of the spectrum.*

The exposition as given in the foregoing has been incomplete as to one important point. Until now we have accepted the idea that the photochemical action investigated was fully reversible. The reversibility as observed in the initial experiments has indeed been a strong argument in favour of the view that the action of the light was confined to some compound directly involved in the light emitting process and did not result in a general disorganisation of the cell.

On the other hand, however, it is well known that ultraviolet radiation has a pronounced lethal effect on all living cells, and therefore it would have been surprising, if such an effect would not also have been manifest in our investigation.

As a matter of fact we soon obtained conclusive evidence for the lethal action of the ultraviolet part of the spectrum also in our experiments, and in this connection we wish to draw the attention of the reader to a curious phenomenon which is observed on studying the recovery of the light emission after the irradiation, and which is clearly shown in Fig. 1 under *C* and *D*.

In this figure under *C* the same part of the luminescent culture as is given under *A* is photographed two hours after the irradiation. It will be seen that in the visible region of the spectrum the inhibition has already largely disappeared, or in other words here the recovery has already made good progress. In the ultraviolet region, however, we are struck by the remarkable phenomenon — already described by GERRETSEN[1]) in 1920 — that the brightness surpasses that of the non-irradiated part of the culture. The reproduction under *D* which brings a photograph taken 6 hours after the irradiation shows that at that time the ultraviolet region has become quite dark, in sharp contrast to what holds for the region of the visible rays where the recovery of the light emission is now quite complete. It is clear that at this time the bacteria in the ultraviolet region have died.

It will not need elucidation that at first sight these observations seem to invalidate the conclusions we have drawn from the experimental results obtained in the ultraviolet region of the spectrum. A closer study of the recovery process of the light emission did, however, show that this was not the case.

In order to study the recovery process a bacterial culture was irradiated during two minutes, using the quartz spectrograph. Then photographs were taken at various moments, *viz.* immediately after the irradiation, and 4, 8, 15, 30, 60 and 120 minutes later. At definite spots in the irradiation spectrum, to wit for 10 wave-lengths in the visible and near ultraviolet region, and for 4 wave-lengths in the ultraviolet region below 300 m μ, the relative intensities of the bacterial light were determined in the usual way, and from these data the course of the recovery of the photochemical effect with time could be computed.

The results are collected in Table II in which the photochemical effects obtained are expressed in an arbitrary measure, for each wave-length taking the effect immediately after the irradiation to be 100.

From Table II it is clear that in the first stages, almost up to a recovery time of 30 minutes, the average figures for the course of the recovery in the visible and near ultraviolet range of the spectrum agree very satisfactorily with the same figures in the short wave ultraviolet range. It is only after that time that in the latter case a quick rise in luminescence takes place leading to the phenomenon earlier described by GERRETSEN.

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[1]) F. C. GERRETSEN, Zentralbl. f. Bakt. II, **52**, 353 (1920).

The results seem to justify the conclusion that the primary photochemical effect, as measured by us immediately after the short time of irradiation, is due to the same cause both in the short wave ultraviolet and in the longer wave range of the spectrum. Apparently the effect, which we may call after its discoverer the GERRETSEN-phenomenon, manifests itself only in the long run and, therefore, does not interfere with our determination of the absorption spectrum.

TABLE II.

Course of the recovery process from photochemical inhibition of the light emission. (For each wave-length the photochemical effect has been expressed in percentage of the effect immediately after the irradiation).

Wave-length in mμ	Time after irradiation in minutes						
	0	4	8	15	30	60	120
436	100	92	82^5	71	51	29^5	11^5
420	100	91	82^5	68^5	49^5	26	10^5
405	100	90	81	69	51	29^5	12
379	100	92	80	66	50^5	25	12^5
366	100	89	81^5	68	49^5	24^5	11
334	100	87	71^5	66	49	—	—
328	100	93^5	78^5	58	50^5	—	—
313	100	82^5	70^5	58	43	—	—
307	100	87^5	76^5	64	53	—	—
302	100	83^5	75	58^5	45	—	—
Average in region >300	100	89	78	65	49	27	11^5
297	100	83	77^5	65^5	46^5	< 0	< 0
292	100	89^5	77	70	55	< 0	< 0
289	100	87	76^5	73^5	62^5	< 0	< 0
280	100	90^5	84	—	—	< 0	< 0
Average in region <300	100	87^5	79	70	55	< 0	< 0

< 0 = increased intensity as compared with non-irradiated zone.

6. *Some remarks regarding the significance of the results obtained.*

The reversible inhibition of bacterial luminescence, as observed in our experiments, leaves practically no doubt that in this phenomenon we are dealing with a photochemical conversion of one of the components of the light emitting system present in the bacteria. It remains, however, questionable whether this conversion is due to a direct absorption of the light quanta by the component in question or whether these quanta are absorbed by some other compound which acts as a *photosensitizer* in the said conversion, thus giving rise to a so-called photochemical reaction of the 2nd type. In the latter case the absorption spectrum determined might relate to a compound which is quite foreign to the light emitting system.

Recent experiences of HARVEY and his school seem indeed to speak in favour of the latter idea. In the Introduction mention has already been made of the fact that in 1925 HARVEY published his observations on the light inhibition of the luminescence of

Cypridina hilgendorfii. In the same publication experiments were reported in which crude luciferin and luciferase solutions were also submitted to irradiation and which clearly showed that only the luciferin was affected by the light. Moreover, it was found that for the conversion of the luciferin the presence of oxygen was indispensable. Recently, however, HARVEY's collaborators CHASE and GIESE [1]) have repeated these experiments, this time working with a luciferin preparation highly purified according to the prescription given by ANDERSON [2]). They now found that contrary to HARVEY's earlier observation the purified luciferin preparation was quite stable in light of wave-lengths longer than 300 m μ. In the short wave ultraviolet (230—280 m μ) luciferin was quickly destroyed, but this destruction differed from that described by HARVEY in its being independent of the presence of oxygen. Besides they found that by adding fluorescent dyes, like eosin, fluorescein or riboflavin, to the solution luciferin could be sensitized to visible light and for this photochemical reaction oxygen proved to be essential. They conclude, therefore, that the photochemical luciferin decomposition observed by HARVEY in his earlier experiments must have been due to the presence of a photosensitizer in the crude extracts, and remark that riboflavin may well have acted as such.

With a view to these results it appeared quite possible that also in the photochemical effect studied by us a photosensitizer would play a role, the more so since ROTTIER [3]) has shortly ago proved that *Ph. phosphoreum* is characterized by a comparatively high content of riboflavin.

However, if riboflavin would have acted as a photosensitizer in our experiments, the absorption spectrum obtained should have been that of riboflavin, and even a superficial comparison of both spectra suffices to reject this idea.

We have, therefore, carefully examined the possibility that our absorption spectrum could be identified with that of some carotenoid, since representatives of this group of compounds have also been encountered as a photosensitizer in physiological processes, for instance in the phototropic reaction of the sporangiophores of *Pilobolus* [4]). But also this hypothesis had to be rejected, since the general character of all known carotenoid spectra differs markedly from that of the absorption spectrum obtained by us.

Now it can, of course, not quite be excluded that in the photochemical effect on the light emission of luminous bacteria some still unknown compound acts as a photosensitizer. However, several arguments are in favour of the idea that in our case we are dealing with a direct photochemical conversion of a component of the light emitting system. So for instance it is difficult to see this conversion as a physiological effect for which a special perception apparatus has been built up in the cell, as may well be the case in the phototropic response of the sporangiophores of *Pilobolus* and related fungi. And the idea that some photosensitizer incidentally present in the cell is responsible for the effect observed can scarcely be reconciled with some additional observations made by us, *viz.*, that the light emission by *Ph. splendidum* and *Ph. Fischeri*, species which widely differ from *Ph. phosphoreum* in several respects, is also inhibited by radiation and that the inactivation spectra in all these cases are closely related.

The most weighty consideration that the absorption spectrum in question is not that of a photosensitizer, but is directly related to some component of the light emitting system is, however, to be found in the results of an as yet unpublished investigation made by A. VAN DER BURG. In this study it has been shown that — contrary to the earlier observations of EYMERS and VAN SCHOUWENBURG [5]) — there exists a slight but unmistakable difference between the spectra of the light emitted by *Ph. phosphoreum* and by *Ph. splendidum*, in so far that the spectrum of the first named species is characterized by a much steeper descent to the side of the short wave-lengths and therefore extends less far in the violet. This result is remarkable, because we have established that the inactivation spectrum of *Ph. phosphoreum* shows a somewhat similar deviation from the otherwise closely related spectrum of *Ph. splendidum* in so far as it extends farther to the side of the longer wave-lengths. Without entering here into a discussion of the interrelationship between emission and absorption spectra, it may be remarked that it can scarcely be incidental that analogous differences exist between the inactivation and emission spectra of both species. This only seems conceivable, if the compound, to which the absorption spectrum is related, is either identical with, or at least closely related to the compound ultimately responsible for the light emission.

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[1]) A. M. CHASE and A. C. GIESE, Journ. Cell. and Comp. Physiol. **16**, 323 (1940).

[2]) R. S. ANDERSON, Journ. Gen. Physiol. **19**, 301 (1935).

[3]) P. B. ROTTIER, Fluorometrische en spectrophotometrische bepaling van lactoflavine in micro-organismen. Diss. Delft (1942).

[4]) E. BÜNNING, Planta **26**, 719 (1937); Ibid. **27**, 148 (1937).

[5]) J. G. EYMERS and K. L. VAN SCHOUWENBURG, Enzymologia **3**, 235 (1937).

Once accepting this idea, it is tempting to make a few suggestions both regarding the role of the compound of which we indirectly determined the absorption spectrum in the mechanism of light emission, and regarding its chemical nature.

Since in the present stage of our investigation these considerations necessarily bear a speculative character, we shall restrict ourselves to a few remarks.

Accepting a close relationship between the mechanism of luminescence both in *Cypridina* and in luminous bacteria, VAN DER KERK[1]) has arrived at the conclusion that this mechanism can be best represented by the following equations:

$$XH_2 + L \rightleftarrows X + LH_2 \qquad (1)$$

$$LH_2 + A \rightleftarrows A \,.\, LH_2 \,. \qquad (2)$$

$$A \,.\, LH_2 + 1/2\, 0_2 \rightarrow A \,.\, LH_2 \,.\, 0 \qquad (3)$$

$$A \,.\, LH_2 \,.\, 0 \rightarrow A \,.\, L_1^* + H_20 \qquad (4)$$

$$A \,.\, L_1^* \rightarrow A \,.\, L_1 + h\nu. \qquad (5)$$

$$A \,.\, L_1 \rightleftarrows A + L_1 \qquad (6)$$

In these equations LH_2 = luciferin, L = dehydroluciferin, L_1 = product of the irreversible (light emitting) oxidation of luciferin, A = luciferase, and * indicates: excited state of a molecule.

Now the results of the investigation of CHASE and GIESE, earlier discussed in this chapter, practically exclude the possibility that either luciferin or luciferase is the photosensitive molecule responsible for the irradiation inhibition observed in our experiments. Maintaining the idea of a close relationship between this molecule and that responsible for the light emission one is, therefore, brought to an identification of our compound with dehydroluciferin (L).

In this connection we wish to point out that the purely chemical investigations of ANDERSON on the chemical nature of luciferin have made it very likely that this compound has the character of a polyphenol, a result which makes it most probable that dehydroluciferin will be some quinone.

Now a fairly recent note by CHAKRAVORLY and BALLENTINE[2]) has brought a most important contribution to our knowledge of the constitution of *Cypridina*-luciferin. These authors have made it acceptable that the polyphenol nucleus bears a side chain of a ketol character which in the light emitting oxidation should be irreversibly oxidized to a carboxyl group with simultaneous production of carbon dioxide.

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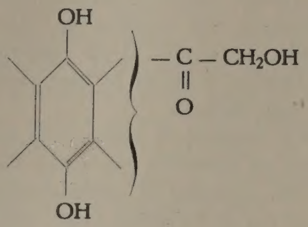
[1]) For the documentation of this scheme we must again refer to the doctorate thesis by VAN DER KERK. The scheme is an elaboration of earlier proposals made by E. N. HARVEY (Erg. d. Enzymforschung **4**, 365 (1935)), by F. H. JOHNSON (Enzymologia **7**, 72 (1939)), by F. H. JOHNSON, K. L. VAN SCHOUWENBURG and A. VAN DER BURG (Enzymologia **7**, 195 (1939)) and by B. CHANCE, E. N. HARVEY, F. H. JOHNSON and G. MILLIKAN (Journ. Cell. and Comp. Physiol. **15**, 195 (1940)).

[2]) P. N. CHAKRAVORLY and R. BALLENTINE, Journ. Amer. Chem. Soc. **63**, 2030 (1941).

Assuming that in bacterial luciferin too the mentioned essential groupings are present, we can also represent it by the partial and schematic formula given by the American authors, *viz.*:



We must, however, emphasize that in this formula the para position of the phenol groups is fully hypothetical.

The question now arises in how far the absorption spectrum determined supports the above formula, and if so, whether it can add to a further identification of the molecule in question.

Although the present state of our knowledge regarding the relation between absorption spectrum and chemical constitution is still far from satisfactory, we may at once conclude that the character of the spectrum in question seems, indeed, to indicate that we are dealing with a compound containing some aromatic ring system. For it has been found that such compounds are all characterized by a pronouncedly steep maximum in the ultraviolet 1)). The general character of our spectrum shows, moreover, a strong resemblance with that of the spectra of naphthalene, anthracene and naphthacene (RADULESCU and collaborators 2))).

The same type of spectrum is also found in various 1.4-naphthoquinone derivatives 3)), whilst the spectra of corresponding compounds of the 1.2-naphthoquinone series are markedly different.

These facts strongly suggest that in dehydroluciferin we are dealing with a compound of the para quinone type, a conclusion which is in agreement with the well-known experience that all such quinones are definitely photosensitive.

The situation of the spectrum, and more especially that of the final absorption to the side of the longer wave-lengths, enables us further to make a choice between the possibilities of a naphthalene, an anthracene and a naphthacene quinone. A critical survey of the spectra of the said hydrocarbons and their derivatives, in so far as these have been determined until now, is strongly in favour of the view that our photosensitive molecule belongs to the naphthoquinone series, and that moreover the $C = O . CH_2OH$ group is directly substituted in the quinone ring. This would bring the dehydroluciferin into the physiologically important class of Vitamine K derivatives, a possibility which is the more attractive, since it has been definitely shown that several bacteria are able to synthesize such compounds 4)).

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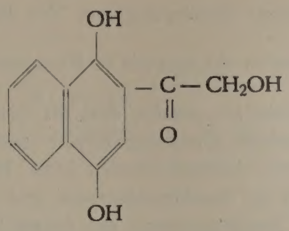
1)) Cf. for instance: K. DIMROTH, Angew. Chemie **52**, 545 (1939).

2)) D. RADULESCU und F. BARBULESCU, Ber. **64**, 2225 (1931); D. RADULESCU und G. OSTROGOVITCH, Ber. **64**, 2233 (1931).

3)) L. F. FIESER, W. P. CAMPBELL and E. M. FRY, Journ. Amer. Chem. Soc. **61**, 2206 (1939).

4)) H. DAM, J. GLAVIND, S. ORLA-JENSEN und A. D. ORLA-JENSEN, Die Naturwissenschaften **29**, 287 (1941) and Zentralbl. f. Bakt. II, **104**, 202 (1941).

The foregoing deductions would,of course, imply that luciferin should either be identical with, or closely related to a compound of the formula:



Although the hypothetical character of this conclusion is fully acknowledged, it seems worth-while to submit it to a more direct test. Accordingly one of our collaborators has made a beginning with the synthesis of several compounds in the naphthoquinone series and with the determination of their absorption spectra.

Utrecht–Delft, Biophysical Research Group, September 1942.

Mathematics. — *Ueber gebundene Semiinvarianten.* Von R. WEITZENBÖCK.

(Communicated at the meeting of November 28, 1942.)

In einer früheren Arbeit[1]) habe ich gezeigt, dass die Einschränkung der Koeffizienten binärer Formen durch semiinvariante Gleichungssysteme auch durch das Nullsetzen einer Reihe von *freien* Semiinvarianten definiert werden kann. Hieraus ergab sich eine semiinvariante Mannigfaltigkeit $\mathfrak{M}$ im Koeffizientenraum und weiters das Zusammenfallen ganzer rationaler gebundener Semiinvarianten mit freien Semiinvarianten auf $\mathfrak{M}$. Ich beweise hier, dass dies auch für *rationale* Semiinvarianten gilt, sodass also auf $\mathfrak{M}$ die freien und die gebundenen Semiinvarianten dieselbe Rationalbasis besitzen.

Gebundene Semiinvarianten binärer Grundformen $f_p, f_q, \ldots$ sind Gradienten L, für die (bei einer Grundform f_p)

$$\Omega L = a_0 \frac{\partial L}{\partial a_1} + 2a_1 \frac{\partial L}{\partial a_2} + \ldots + p a_{p-1} \frac{\partial L}{\partial a_p} = 0 \quad . \quad . \quad . \quad (1)$$

gilt für alle Koeffizienten a_i, die ein semiinvariantes Gleichungssystem

$$S_1 = 0, \; S_2 = 0, \ldots, S_h = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

befriedigen. Das System (2) bestimmt im Koeffizientenraum eine algebraische Mannigfaltigkeit $\mathfrak{M}$. Man kann daher auch kurz sagen: $\Omega L = 0$ auf $\mathfrak{M}$.

Nach dem HILBERTschen Nullstellensatz gilt daher für einen gewissen Exponenten m:

$$(\Omega L)^m = U_1 S_1 + U_2 S_2 + \ldots + U_h S_h \quad . \quad . \quad . \quad . \quad (3)$$

identisch in den a_i, wobei die U_i Formen der a_i sind.

Geht man zur Rationalbasis der Protomorphen über, so lässt sich (2) ersetzen durch

$$J_1^* = 0, \; J_2^* = 0, \ldots, J_h^* = 0$$

mit freien Semiinvarianten J_i^* und statt (3) haben wir

$$\left(\frac{\partial L^*}{\partial a_1} \right)^m = V_1^* J_1^* + \ldots + V_h^* J_h^* , \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

wobei die V_i^* jetzt Polynome der $a_1, A_2, A_3, \ldots, A_p$ sind.

Sei jetzt

$$L = \frac{P}{Q}, \; L^* = \frac{P^*}{Q^*}, \; \Omega(L) = \frac{Q\,\Omega(P) - P\,\Omega(Q)}{Q^2} \quad . \quad . \quad . \quad (5)$$

eine rationale gebundene Semiinvariante, d.h. ein Bruch, für den auf $\mathfrak{M}$ $\Omega(L) = 0$ gilt. Dies gibt wie bei (4):

$$\left(Q^* \frac{\partial P^*}{\partial a_1} - P^* \frac{\partial Q^*}{\partial a_1} \right)^m = \Sigma\, V_i^* J_i^* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Wir entwickeln P^* und Q^* nach Potenzen von a_1, setzen also

$$P^* = P_0^* + \frac{a_1}{1!} P_1^* + \ldots \qquad \frac{\partial P^*}{\partial a_1} = P_1^* + \frac{a_1}{1!} P_2^* + \ldots$$

$$Q^* = Q_0^* + \frac{a_1}{1!} Q_1^* + \ldots \qquad \frac{\partial Q^*}{\partial a_1} = Q_1^* + \frac{a_1}{1!} Q_2^* + \ldots \; .$$

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[1]) Proc. Kon. Akad. v. Wetensch., Amsterdam, **38**, 24—29 (1935).

Dann gibt (6)

$$(Z_0^* + a_1 Z_1^* + \dots)^m = \Sigma V_i^* J_i^* \qquad \text{(identisch in allen } a_i\text{)} \quad . \quad . \quad (7)$$

mit

$$\left.\begin{aligned} Z_r^* &= \frac{1}{0!\,r!}(Q^* P^*)_{0,\,r+1} + \frac{1}{1!\,(r-1)!}(Q^* P^*)_{1,\,r} + \dots + \frac{1}{r!\,0!}(Q^* P^*)_{r,\,1} \\ (Q^* P^*)_{ik} &= -(Q^* P^*)_{ki} = Q_i^* P_k^* - Q_k^* P_i^* \end{aligned}\right\} \quad (8)$$

oder ausführlicher:

$$Z_0^* = (Q^* P^*)_{01}\,, \quad Z_1^* = (Q^* P^*)_{02}$$

$$Z_2^* = \frac{1}{2!}(Q^* P^*)_{03} + \left(\frac{1}{1!^2} - \frac{1}{2!}\right)(Q^* P^*)_{12}$$

$$Z_3^* = \frac{1}{3!}(Q^* P^*)_{04} + \left(\frac{1}{1!\,2!} - \frac{1}{3!}\right)(Q^* P^*)_{13} \quad \text{u.s.f.}$$

In (7) rechts sind die a_1 nur in den V_i^* enthalten. Daraus folgt, dass Z_0^*, Z_1^*, ... auf $\mathfrak{M}$ Null sind. Hieraus ergibt sich weiter aus (8), dass alle Ausdrücke $(Q^*P^*)_{ik}$, also insbesondere

$$(Q^* P^*)_{0i} = Q_0^* P_i^* - Q_i^* P_0^*$$

auf $\mathfrak{M}$ verschwinden. D.h. auf $\mathfrak{M}$ gilt

$$Q_i^* = \frac{Q_0^*}{P_0^*} P_i^* .$$

Setzen wir dies in

$$Q^* = Q_0^* + \frac{a_1}{1!} Q_1^* + \frac{a_1^2}{2!} Q_2^* + \dots$$

ein, so folgt

$$Q^* = \frac{Q_0^*}{P_0^*} P^*, \ \frac{P}{Q^*} = \frac{P_0^*}{Q_0^*} \qquad \text{(auf } \mathfrak{M}\text{)} .$$

P_0^* und Q_0^* sind ohne a_1, also freie Semiinvarianten und wir haben: Auf $\mathfrak{M}$ fällt jede rationale gebundene Semiinvariante mit einer freien Semiinvariante zusammen. Oder auch: *die gebundenen Semiinvarianten haben auf $\mathfrak{M}$ dieselbe Rationalbasis wie die freien Semiinvarianten.*

Physics. — *Sur le comportement d'un ensemble canonique lors d'une transformation adiabatique.* Par L. ROSENFELD. (Communicated by Prof. H. A. KRAMERS.)

(Communicated at the meeting of November 28, 1942.)

On sait que les conceptions développées par GIBBS dans son célèbre traité de Mécanique statistique (1) ont fait l'objet d'une critique pénétrante de la part de P. et T. EHRENFEST (2). Pour ce qui est de la manière générale dont une répartition non stationnaire tend vers la répartition d'équilibre statistique, les récents progrès de la théorie ergodique (3) permettent actuellement de présenter les idées de GIBBS sous une forme qui soit à l'abri de ces critiques. Mais dans la discussion de l'effet de transformations quasistatiques sur une répartition d'équilibre, il y a un point qui, à ma connaissance, est encore resté dans l'ombre. Tandis que la considération de transformations isothermes ne présente évidemment aucune difficulté, il n'en est pas tout-à-fait de même des transformations adiabatiques. Si, partant d'un ensemble distribué canoniquement, on fait subir aux paramètres extérieurs une variation brusque infiniment petite, après quoi on maintient les paramètres à leurs nouvelles valeurs, les systèmes de l'ensemble restant thermiquement isolés, on doit s'attendre — pour pouvoir établir en toute généralité l'analogue statistique du concept d'entropie — à ce que la nouvelle répartition d'équilibre vers laquelle „tend" l'ensemble considéré (au sens de la théorie ergodique) soit de nouveau une répartition canonique, correspondant aux nouvelles valeurs des paramètres extérieurs et à une valeur convenablement modifiée du module. P. et T. EHRENFEST observent avec raison (4) que cette propriété, admise implicitement par GIBBS lorsqu'il discute les analogies thermodynamiques au Chapitre XIV de son livre, n'est pas démontrée, ni même énoncée, dans les chapitres précédents. LORENTZ (5), dans l'exposé très clair qu'il donne de la théorie de GIBBS, glisse trop rapidement sur cette question; il se contente d'invoquer sommairement l'équivalence, pour des systèmes à un grand nombre de degrés de liberté, d'une répartition canonique et d'une répartition microcanonique. Il est clair que dans ce cas, ce genre de considérations est insuffisant; il importe en effet de savoir si la répartition finalement atteinte serait ou non modifiée si on mettait ensuite les systèmes de l'ensemble considéré en contact avec un thermostat ayant pour température celle qu'il faut, d'après la thermodynamique, attribuer aux systèmes après la transformation adiabatique: car une telle modification, qui aurait lieu si la répartition finalement atteinte n'était pas canonique, signifierait — en contradiction avec la thermodynamique — une augmentation d'entropie lors de ce contact.

Naturellement, le problème qui nous occupe a une solution bien déterminée, qui doit se présenter d'elle-même dès qu'on analyse le processus considéré à l'aide des méthodes de GIBBS. C'est ce que nous nous proposons de montrer brièvement ici. Nous partons donc d'une répartition canonique de module θ_0; nous n'envisagerons qu'un seul paramètre extérieur, dont la valeur initiale sera a_0. La densité en phase sera donc donnée initialement par $e^{H(p,q;\theta_0,a_0)}$ avec

$$H(p,q;\theta_0,a_0)=\frac{F(\theta_0,a_0)-E(p,q;a_0)}{\theta_0}, \quad \ldots\ldots \quad (1)$$

F représentant l'énergie libre, E l'énergie dans la phase[1]) (p,q) d'un système de l'ensemble. Si le paramètre extérieur subit la variation brusque da et garde ensuite la valeur $a=a_0+da$, la répartition précédente ne sera tout d'abord pas modifiée, mais les „surfaces" d'énergie constante dans l'extension en phase seront déformées, de sorte que la répartition ne sera plus uniforme dans les couches comprises entre deux surfaces infiniment voisines $E(a)$, $E(a)+dE$. Comme les systèmes de l'ensemble restent énergétiquement isolés, cette répartition, en vertu du théorème ergodique, évoluera dans chacune de ces couches séparément vers une répartition uniforme; la densité de celle-ci n'est autre chose que la moyenne microcanonique, pour la couche en question, de la répartition initiale. Mais

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[1]) Les lettres p, q désignent les coordonnées $p_1, p_2, \ldots, p_f$; $q_1, q_2, \ldots, q_f$ de la phase

$$e^{H(p,q;\theta_0,a_0)} = e^{H(p,q;\theta_0,a)}\left\{1-\frac{da}{\theta_0}\left(\frac{\partial F}{\partial a}-\frac{\partial E}{\partial a}\right)\right\}$$

$$= e^{H(p,q;\theta_0,a)}\left\{1-\frac{da}{\theta_0}\left(A-\bar{A}\right)\right\},$$

en appelant $A=-\frac{\partial E}{\partial a}$ la force généralisée relative au paramètre a et $\bar{A}$ sa valeur moyenne dans la répartition canonique initiale 1)). En observant que $e^{H(p,q;\theta_0,a)}$ et $\bar{A}$ sont constants dans la couche $\{E(a),\ E(a)+dE\}$, on obtient donc pour la répartition finale une densité

$$e^{H(p,q;\theta_0,a)}\left\{1-\frac{da}{\theta_0}\left(\left.\overline{A}\right]_{E(a)}-\bar{A}\right)\right\}, \quad . \ . \ . \ . \ . \ . \quad (2)$$

le symbole $\left.\overline{A}\right]_{E(a)}$ désignant la moyenne microcanonique de A pour l'énergie $E(a)$. Il s'agit maintenant de comparer cette répartition à une répartition canonique $e^{H(p,q;\,\theta,a)}$ de module $\theta=\theta_0+d\theta$ convenablement choisi.

Si nous nous limitons au cas de systèmes à un grand nombre de degrés de liberté (le seul qui permette une comparaison avec la thermodynamique), il suffira d'étudier la répartition (2) au voisinage de l'énergie moyenne $\bar{E}$ (La barre désigne toujours la moyenne canonique). Nous écrirons donc, la valeur du paramètre a restant fixe,

$$\left.\overline{A}\right]_E = \left.\overline{A}\right]_{\bar{E}} + \frac{\partial \left.\overline{A}\right]_{\bar{E}}}{\partial \bar{E}}\left(E-\bar{E}\right) + \frac{1}{2}\,\frac{\partial^2 \left.\overline{A}\right]_{\bar{E}}}{\partial \bar{E}^2}\left(E-\bar{E}\right)^2$$

et

$$\bar{A} = \left.\overline{A}\right]_{\bar{E}} + \frac{1}{2}\,\frac{\partial^2 \left.\overline{A}\right]_{\bar{E}}}{\partial \bar{E}^2}\,\overline{\left(E-\bar{E}\right)^2},$$

d'où nous tirons pour la répartition (2)

$$e^{H(p,q;\theta_0,a)}\left\{1-\frac{da}{\theta_0}\,\frac{\partial \left.\overline{A}\right]_{\bar{E}}}{\partial \bar{E}}\left(E-\bar{E}\right)-\frac{1}{2}\,\frac{da}{\theta_0}\,\frac{\partial^2 \left.\overline{A}\right]_{\bar{E}}}{\partial \bar{E}^2}\left[\left(E-\bar{E}\right)^2-\overline{\left(E-\bar{E}\right)^2}\right]\right\}.$$

Définissons maintenant $d\theta$ par

$$\frac{d\theta}{\theta_0} = -da\,\frac{\partial \left.\overline{A}\right]_{\bar{E}}}{\partial \bar{E}}; \quad . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

nous pouvons alors remplacer

$$e^{H(p,q;\theta_0,a)}\left\{1+\frac{d\theta}{\theta_0^2}\left(E-\bar{E}\right)\right\}$$

par

$$e^{H(p,q;\theta,a)} \quad (\text{avec } \theta=\theta_0+d\theta).$$

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1) Si l'on se borne aux termes du premier ordre en da, il est permis de remplacer a_0, par a dans les fonctions H et $\bar{A}$ intervenant dans le coefficient de da.

puisque

$$\frac{\partial}{\partial \theta_0}\left(\frac{F}{\theta_0}\right) = -\frac{\overline{E}}{\theta_0^2}.$$

La répartition (2) finalement atteinte après la transformation adiabatique infinitésimale envisagée prend donc la forme

$$e^{H(p,q;\theta,a)}\left\{1 - \frac{1}{2}\frac{da}{\theta_0}\frac{\partial^2 \overline{A}\big|_{\overline{E}}}{\partial \overline{E}^2}\left[(E-\overline{E})^2 - \overline{(E-\overline{E})^2}\right]\right\}; \quad . \quad . \quad (4)$$

elle diffère donc par le dernier facteur de la répartition canonique de module θ, relative à la valeur finale a du paramètre variable.

Il est toutefois facile de voir que ce facteur ne joue qu'un rôle négligeable pour des systèmes à un grand nombre de degrés le liberté [1]). La correction apportée à la moyenne canonique d'une grandeur quelconque G est en effet (en confondant ici, ce qui est permis, θ et θ_0)

$$-\frac{1}{2}\frac{da}{\theta}\frac{\partial^2 \overline{A}\big|_{\overline{E}}}{\partial \overline{E}^2}\overline{(G-\overline{G})(E-\overline{E})^2},$$

ou encore, grâce à une formule générale de GIBBS [2]),

$$-\frac{1}{2}\frac{da}{\theta}\frac{\partial^2 \overline{A}\big|_{\overline{E}}}{\partial \overline{E}^2}\,\theta^2\frac{\partial}{\partial \theta}\left(\theta^2\frac{\partial \overline{G}}{\partial \theta}\right);$$

on voit que cette quantité, lorsque le nombre de degrés de liberté augmente, tend vers zéro encore plus rapidement que les fluctuations moyennes de l'énergie (et, sauf circonstances exceptionnelles, des autres grandeurs) dans l'ensemble canonique. Il en est d'ailleurs de même de la correction apportée par le facteur en question aux fluctuations moyennes des grandeurs de l'ensemble. Nous arrivons donc à la conclusion que la répartition finale se réduit bien à une répartition canonique.

Retournons à la formule (3), qui donne la variation du module. Nous pouvons y remplacer $\frac{\partial \overline{A}\big|_{\overline{E}}}{\partial \overline{E}}$ par $\frac{\partial \overline{A}}{\partial \overline{E}}$, c.à.d. $\frac{\partial \overline{A}}{\partial \theta}\Big/\frac{\partial \overline{E}}{\partial \theta}$; elle prend alors la forme bien connue de l'équation thermodynamique des transformations adiabatiques, déduite de la constance de l'entropie.

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3. Voir p. ex. E. HOPF, Ergodentheorie, Ergebn. d. math. Wiss. V, 2 (1937).
4. P. u. T. EHRENFEST, loc. cit. § 23*b*.
5. H. A. LORENTZ, Ueber den zweiten Hauptsatz der Thermodynamik, § 81 (Abhandlungen über theoretische Physik I, 1907, S. 292).

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[1]) Dans le cas d'un gaz parfait, d'un champ de rayonnement pur, ou d'un autre système pour lequel $\frac{\partial^2 \overline{A}\big|_{\overline{E}}}{\partial \overline{E}^2} = 0$, on aurait à considérer un facteur dépendant de puissances encore plus élevées de $E-\overline{E}$, qui serait *a fortiori* négligeable.

[2]) Il s'agit de la formule (257) (Chapitre VII, éd. franç. p. 81). Il suffit de l'écrire pour $h = 1$ et $h = 2$ et de combiner convenablement les formules obtenues; la grandeur G est ici supposée *ne pas* dépendre du module.

Mathematics. — *Ueber die Existenz der Eigenfunktionen eines symmetrisierbaren Kernes.* Von A. C. ZAANEN. (Communicated by Prof. W. VAN DER WOUDE.)

(Communicated at the meeting of November 28, 1942.)

1. Einleitung.

Herr O. D. KELLOGG hat einen eleganten Beweis dafür gegeben, dass jeder symmetrische Kern $K(s,t)$ $(a \leqq s, t \leqq b)$ mindestens einen Eigenwert besitzt 1). Es wurde dabei von $K(s,t)$ ausser der Symmetrie noch vorausgesetzt, dass $\int K^2(s,t)dt \not\equiv 0$ 2) und $\leqq B^2$ und dass bei jedem $\varepsilon > 0$ ein $\delta > 0$ zu finden sei, sodass

$$\left| \int [K(s_1, t) - K(s_2, t)] \, \psi(t) \, dt \right| < \varepsilon$$

gilt für $| s_1 - s_2 | < \delta$, wenn $\psi(t)$ eine normierte, aber übrigens willkürliche kwadratisch integrierbare Funktion ist.

Nehmen wir an, dass $K(s,t)$ in s (und also auch in t) kwadratisch integrierbar ist und dass

$$\lim_{s_1 \to s_2} \int [K(s_1, t) - K(s_2, t)]^2 \, dt = 0$$

ist, so genügt dieser Kern den obengenannten Bedingungen.

Wir werden jetzt zeigen, dass die Beweismethode sich ausbreiten lässt auf einige Kerne, die nicht länger symmetrisch zu sein brauchen, sondern gehören zur Klasse der sog. symmetrisierbaren Kerne. Um zu definieren, was ein symmetrisierbarer Kern ist, erinnern wir daran, dass ein symmetrischer Kern $H(s,t)$ positiv-definit genannt wird, wenn

$$\int\int H(s,t) \, f(s) \, f(t) \, ds \, dt \geqq 0$$

ist für jedes kwadratisch integrierbare $f(s)$. Gibt es nun ein symmetrisches und positiv-definites $H(s,t)$, sodass

$$P(s,t) = \int H(s,\tau) \, K(\tau,t) \, d\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

symmetrisch ist, so heisst $K(s,t)$ (links-) symmetrisierbar.

Zu den wichtigsten Beispielen gehören

$$1^0. \qquad K(s,t) = A(s) \, H(s,t), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

wobei $A(s)$ messbar und beschränkt, $H(s,t)$ symmetrisch und positiv-definit ist. Dieser Kern gibt Anlass zu einer sog. Integralgleichung dritter Art.

$$2^0. \qquad K(s,t) = \int A(s,\tau) \, H(\tau,t) \, d\tau, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

wobei $A(s,t)$ symmetrisch, $H(s,t)$ wieder symmetrisch und positiv-definit ist.

Man sieht sofort ein, dass in beiden Fällen die durch (1) definierte Funktion $P(s,t)$ wirklich symmetrisch ist.

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1) O. D. KELLOGG, On the existence and closure of sets of characteristic functions, Math. Annalen 86 (1922), S. 14—17.

2) Jedes Integral, bei dem keine Grenzen angeschrieben sind, ist als $\int_a^b$ zu lesen.

Im Folgenden werden wir noch die Voraussetzung machen, dass aus

$$\int H(s,t)\,f(t)\,dt \equiv 0 \quad \ldots\ldots\ldots \quad (4)$$

für ein stetiges $f(t)$ folgt

$$\int K(s,t)\,f(t)\,dt \equiv 0.$$

Ist $H(s,t)$ abgeschlossen, d.h. hat (4) $f(t)\equiv 0$ zur Folge, so genügt $K(s,t)$ automatisch dieser Bedingung. Dasselbe gilt, wenn $K(s,t)$ durch (2) oder (3) definiert ist. Wir werden beweisen, dass nun $K(s,t)$ immer einen Eigenwert besitzt, d.h. also, dass es eine (reelle) Zahl λ gibt, wofür

$$\varphi(s) = \lambda \int K(s,t)\,\varphi(t)\,dt$$

eine nicht identisch verschwindende Lösung $\varphi(s)$ hat.

Wir verwenden die Bezeichnungen

$$(f,g) = \int f(s)\,g(s)\,ds$$

$$Hf = \int H(s,t)\,f(t)\,dt, \qquad Kf = \int K(s,t)\,f(t)\,dt,$$

$$Pf = \int P(s,t)\,f(t)\,dt, \qquad K^{*}f = \int K(t,s)\,f(t)\,dt,$$

$$\|f\|^2 = (Hf,f) \quad , \qquad |||f|||^2 = (f,f) \quad .$$

Die Symmetrie von H und P wird dadurch zum Ausdruck gebracht, dass $(Hf,g) = (f,Hg)$ und $(Pf,g) = (f,Pg)$. Die gemachte Voraussetzung über H und K lässt sich jetzt so formulieren: Aus $Hf=0$ für ein stetiges f folgt $Kf=0$.

2. Vorbereitungen.

Um den Beweis nicht zu unterbrechen, lassen wir einige Bemerkungen vorangehen.

1°. Eine abzählbar unendliche Folge $\{f_n(s)\}$ heisst gleichgradig stetig, wenn bei jedem $\varepsilon>0$ ein $\delta>0$ zu finden ist, sodass $|f_n(s_1)-f_n(s_2)|<\varepsilon$ gilt für alle n, wenn nur s_1 und s_2 im Intervall $a\leqq s\leqq b$ liegen und $|s_1-s_2|<\delta$ ist. Ist die Folge auch noch gleichmässig beschränkt, so enthält sie eine gleichmässig konvergente Teilfolge (Satz von ARZELÀ).

2°. Von den Kernen $H(s,t)$ und $K(s,t)$ wollen wir (abgesehen von dem Fall, wo $K(s,t)$ durch (2) definiert ist) voraussetzen, dass sie in s und t kwadratisch integrierbar sind und dass

$$\int K^2(t,s)\,dt \leqq B^2,$$

$$\lim_{s_1\to s_2} \int [H(s_1,t)-H(s_2,t)]^2\,dt = 0, \qquad \lim_{s_1\to s_2} \int [K(s_1,t)-K(s_2,t)]^2\,dt = 0$$

ist. Es ist leicht einzusehen, dass dann $\int H^2(s,t)dt$ und $\int K^2(s,t)dt$ stetig, also beschränkt sind. Sind für die Folge $\{f_n(s)\}$ die Normen $|||f_n||| = (f_n, f_n)^{\frac{1}{2}}$ beschränkt, so ist die Folge $\{g_n(s)\}$, wobei $g_n = Hf_n$, gleichmässig beschränkt und gleichgradig stetig, wie eine Anwendung der SCHWARZschen Ungleichung sofort erkennen lässt; sie enthält also nach dem Satz von ARZELÀ eine gleichmässig konvergente Teilfolge $\{g_j(s)\}$ $(j = n_1, n_2, \ldots)$. Wenn $K(s,t)$ statt $H(s,t)$ eingesetzt wird, so gilt dasselbe, auch noch in dem Sonderfall $K(s,t) = A(s)\,H(s,t)$. Ist nämlich $\{g_j(s)\}$ die Teilfolge aus $\{g_n(s)\}$, worüber schon gesprochen wurde, so konvergiert auch $\{A(s)g_j(s) = Kf_j\}$ wegen der Beschränktheit von $A(s)$ gleichmässig.

3°. Ist $\{\lambda_n\}$ $(n = 1, 2, \ldots)$ die Folge der Eigenwerte eines symmetrischen, positiv-definiten Kernes $H(s, t)$, $\{\varphi_n(s)\}$ die Folge der zugehörigen normierten Eigenfunktionen, $f(s)$ eine kwadratisch integrierbare Funktion und $a_n = (f, \varphi_n)$ für alle n, so ist, wie bekannt

$$Hf = \sum_{n=1}^{\infty} a_n \lambda_n^{-1} \varphi_n(s) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

und diese Reihe konvergiert gleichmässig. Also gilt

$$\| f \|^2 = (Hf, f) = \Sigma a_n^2 \lambda_n^{-1}.$$

Hieraus erkennt man, dass alle Eigenwerte positiv sind. Wir nehmen an, dass $0 < \lambda_1 \leqq \lambda_2 \leqq \ldots$

4°. Ist $H_2(s, t) = \int H(s, \tau)\, H(\tau, t) d\tau$, so hat bekanntlich $H_2(s, t)$ die Eigenwerte λ_n^2 und dieselben Eigenfunktionen wie $H(s, t)$. Also ist

$$(H_2 f, f) = \Sigma a_n^2 \lambda_n^{-2} = \Sigma \lambda_n^{-1} a_n^2 \lambda_n^{-1} \leqslant \lambda_1^{-1} \Sigma a_n^2 \lambda_n^{-1} = \lambda_1^{-1} \| f \|^2 \quad . \quad (6)$$

Weiter weisen wir daraufhin, dass aus $\| f \| \neq 0$ folgt $Hf \not\equiv 0$ und umgekehrt (denn $Hf \not\equiv 0$ hat wegen (5) zur Folge, dass nicht alle $a_n = 0$ sein können).

5°. Zum Schluss sei noch erwähnt, dass aus $\| f + \lambda g \| \leqq 0$ für jedes λ ein Analogon der SCHWARZschen Ungleichung folgt, nämlich

$$|(Hf, g)| \leqslant \| f \| \cdot \| g \| \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

3. Der Beweis.

Wir nehmen an, dass für die in (1) definierte Funktion $P(s, t)$ gilt $\int P^2(s, t) dt \not\equiv 0$. Es gibt also ein s_0 mit $\int P(s_0, t)\, P(t, s_0) dt \neq 0$. Wenn s in einer Umgebung von s_0 liegt, ist mithin auch $\int P(s, t)\, P(t, s_0) dt \neq 0$. Setzen wir $P(t, s_0) = f_0(t)$, so haben wir damit eine kwadratisch integrierbare Funktion gefunden, wofür $Pf_0 \not\equiv 0$ ist. Aus $Pf_0 = HKf_0 = K^*Hf_0 \not\equiv 0$ folgt $Hf_0 \not\equiv 0$, also $\| f_0 \| \neq 0$ nach Bemerkung 4°.

Für $n = 0, 1, 2, \ldots$ setzen wir nun $f_{n+1} = K\bar{f}_n$, wobei $\bar{f}_n(s) = \| f_n \|^{-1} f_n(s)$. Soll diese Definition einen Sinn haben, so müssen wir zeigen, dass alle $\| f_n \| \neq 0$ sind. Für $\| f_0 \|$ ist das geschehen und $\| f_1 \| \neq 0$ ist eine Folge von $Hf_1 = HK\bar{f}_0 = P\bar{f}_0 \not\equiv 0$. Es sei also schon bewiesen, dass $\| f_i \| \neq 0$ sei für $i \leqq n$. Dann ist wegen der Symmetrie von $P(s, t)$ und (7)

$$\| f_n \| = (Hf_n, \bar{f}_n) = (HK\bar{f}_{n-1}, \bar{f}_n) = (P\bar{f}_{n-1}, \bar{f}_n) = (\bar{f}_{n-1}, P\bar{f}_n) = (\bar{f}_{n-1}, HK\bar{f}_n) =$$
$$= (\bar{f}_{n-1}, Hf_{n+1}) = (H\bar{f}_{n-1}, f_{n+1}) \leqslant \| \bar{f}_{n-1} \| \cdot \| f_{n+1} \| = \| f_{n+1} \|.$$

Die Folge $\{\| f_n \|\}$ ist also nicht-abnehmend, sodass $\| f_n \| \neq 0$ allgemein gilt. Wir bemerken noch, dass sich

$$(Hf_{n-1}, f_{n+1}) = \| f_{n-1} \| \cdot \| f_n \| \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

ergibt aus der Gleichheit $\| f_n \| = (H\bar{f}_{n-1}, f_{n+1})$, die bei der soeben ausgeführten Berechnung auftrat.

Jetzt wollen wir zeigen, dass die Folge $\{\bar{f}_n(s)\}$ eine Teilfolge $\{\bar{f}_i(s)\}$ $(i = n_1, n_2, \ldots)$ von beschränkter Norm $||| \bar{f}_i |||$ enthält. Es ist nach (6)

$$||| H\bar{f}_n |||^2 = (H\bar{f}_n, H\bar{f}_n) = (H_2 \bar{f}_n, \bar{f}_n) \leqslant \lambda_1^{-1} \| \bar{f}_n \|^2 = \lambda_1^{-1},$$

also

$$|g_{n+1}(s)| = |Hf_{n+1}| = |HKf_n| = |K^* H\bar{f}_n| \leqslant [\int K^2(t,s)\,dt]^{\frac{1}{2}} . |||Hf_n||| \leqslant B\lambda_1^{\frac{1}{2}} .$$

Die Folge $\{g_n(s)\}$ ist mithin gleichmässig beschränkt.

Nehmen wir nun an, dass jede Teilfolge aus $\{|||\bar{f}_n|||\}$ unbeschränkt sei, also $\lim\limits_{n\to\infty} |||\bar{f}_n||| = \infty$. Es gibt dann unendlich viele n, wofür $|||\bar{f}_{n+2}||| \geqslant |||\bar{f}_n|||$; sonst gäbe es nämlich ein N, sodass für $n \geqslant N$

$$|||\bar{f}_n||| < max \{|||\bar{f}_N|||, |||\bar{f}_{N+1}|||\},$$

also wäre $\{|||f_n|||\}$ beschränkt.

Es sei also $\{|||\bar{f}_j|||\}$ eine Teilfolge mit $|||f_{j+2}||| \geqslant |||\bar{f}_j|||$. Nach Bemerkung 2°. enthält $\{|||f_j|||^{-1} f_{j+1}(s) = K|||\bar{f}_j|||^{-1}\bar{f}_j\}$ eine Teilfolge $\{|||\bar{f}_k|||^{-1} f_{k+1}(s)\}$, die gleichmässig konvergiert gegen eine stetige Funktion $f(s)$. Es gilt

$$|||\bar{f}_k|||^{-1} g_{k+1}(s) = H|||\bar{f}_k|||^{-1} f_{k+1}$$

und aus der Beschränktheit der $g_{k+1}(s)$, zusammen mit $\lim\limits_{k\to\infty} |||\bar{f}_k||| = \infty$ folgt nun

$$0 = Hf.$$

Weiter ist

$$|||\bar{f}_k|||^{-1} . ||f_{k+1}||\, f_{k+2}(s) = K|||\bar{f}_k|||^{-1} f_{k+1},$$

also konvergiert $|||\bar{f}_k|||^{-1} . ||f_{k+1}||\, f_{k+2}(s)$ gleichmässig gegen $g(s) = Kf$. Es ist $|||g||| \geqslant ||f_1||^2$ wegen $||f_n|| \geqslant ||f_1||$ und $|||\bar{f}_{k+2}||| \geqslant |||\bar{f}_k|||$, mithin $g(s) \neq 0$ entgegen unsrer Annahme, dass aus $Hf = 0$ folgen würde $Kf = 0$.

Es gibt also eine Teilfolge $\{f_i(s)\}$ von beschränkter Norm $|||\bar{f}_i|||$. Wie Bemerkung 2°. lehrt, enthält dann $\{f_{i+1}(s) = K\bar{f}_i\}$ eine gleichmässig gegen eine Funktion $\alpha(s)$ konvergierende Teilfolge $\{f_l(s)\}$. Weiter ist die nicht abnehmende Folge $\{||f_n||\}$ nun gleichfalls beschränkt, also besteht

$$\lim_{n\to\infty} ||f_n|| = \lambda^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Man sieht leicht ein, dass wegen $\lim\limits_{l\to\infty} \bar{f}_l(s) = \lambda\alpha(s)$ auch die Teilfolgen $f_{l+1}(s)$ und $f_{l+2}(s)$ gleichmässig konvergieren. Es sei demzufolge

$$\lim_{l\to\infty} f_l(s) = \alpha(s), \quad \lim_{l\to\infty} f_{l+1}(s) = \beta(s), \quad \lim_{l\to\infty} f_{l+2}(s) = \gamma(s) \quad . \quad . \quad (10)$$

Wir finden dann, unter Benutzung von (8) und (9)

$$||\alpha - \gamma||^2 = \lim_{l\to\infty} ||f_l - f_{l+2}||^2 = \lim_{l\to\infty} [||f_l||^2 - 2(Hf_l, f_{l+2}) + ||f_{l+2}||^2] =$$

$$= \lim_{l\to\infty} [||f_l||^2 - 2||f_l|| . ||f_{l+1}|| + ||f_{l+2}||^2] = 0,$$

also ist

$$H(\alpha - \gamma) = 0,$$

mithin nach unsrer Annahme auch

$$K(\alpha - \gamma) = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Aus (9), (10) und (11) folgt jetzt

$$\beta(s) = \lambda K \gamma,$$
$$\gamma(s) = \lambda K \beta.$$

Hieraus ist aber sofort zu ersehen, dass $\beta(s) + \gamma(s)$ und $\beta(s) - \gamma(s)$ Eigenfunktionen von $K(s, t)$ sind, gehörend zu den Eigenwerten λ bezw. $-\lambda$. Wären beide $\equiv 0$, so wäre auch $\beta(s) \equiv 0$, entgegen $\| \beta \| = \lim_{l \to \infty} \| f_{l+1} \| = \lambda^{-1}$.

Mindestens eine der Gleichungen

$$\varphi(s) = \lambda \int K(s, t)\, \varphi(t)\, dt,$$
$$\varphi(s) = -\lambda \int K(s, t)\, \varphi(t)\, dt$$

hat also eine nicht identisch verschwindende Lösung $\varphi(s)$.

Wenn wir die so gefundende Eigenfunktion $\varphi_1(s)$ nennen, können wir uns die Frage vorlegen, ob ausser $\varphi_1(s)$ noch weitere Eigenfunktionen bestehen. Gibt es nun eine Funktion $g(s)$, wofür $Pg \not\equiv 0$ und $(H\varphi_1, g) = 0$ ist, so können wir, von dieser $g(s)$ ausgehend (sie übernimmt die Rolle von $f_0(s)$) in der schon gezeigten Weise eine zweite Eigenfunktion $\varphi_2(s)$ bekommen. $\varphi_2(s)$ ist von $\varphi_1(s)$ verschieden, denn $(H\varphi_1, \varphi_2) = 0$, wovon man sich leicht überzeugt. Gibt es dann noch eine $h(s)$ mit $Ph \not\equiv 0$, $(H\varphi_1, h) = (H\varphi_2, h) = 0$, so besteht eine dritte Eigenfunktion $\varphi_3(s)$. In dieser Weise können wir weitergehen.

Ist $\{\varphi_n(s)\}$ das vollständige System der Eigenfunktionen und ist $(H\varphi_n, \psi) = 0$ für eine Funktion $\psi(s)$ und alle n, so muss $P\psi \equiv 0$ sein, denn wäre $P\psi \not\equiv 0$, so gäbe es ausser den $\{\varphi_n(s)\}$ noch eine Eigenfunktion.

Mathematics. — *Zur Theorie des Maszes im Körper der P-adischen Zahlen.* Von A. F. MONNA. (Communicated by Prof. W. VAN DER WOUDE.)

(Communicated at the meeting of November 28, 1942.)

1. TURKSTRA hat im Körper der P-adischen Zahlen eine Masztheorie entwickelt, analog dem LEBESGUEschen Masz im Körper der reellen Zahlen [1]). Ich möchte hier auf einen einfachen Verband zwischen diesen beiden Maszen hinweisen.

Es sei R die Menge der nicht negativen reellen Zahlen $\leqq 1$ und O eine offene Menge in R. O ist die Summe höchstens abzählbar unendlich vieler abgeschlossenen Intervallen I_i die höchstens Randpunkte gemeinschaftlich haben:

$$O = \sum_{i=1}^{\infty} I_i .$$

Es sei a_i der linke Randpunkt von I_i und ε_i die Länge von I_i. Wir schreiben a_i als Potenzreihe in P^{-1} (P prim)

$$a_i = \sum_{j=0}^{\infty} \lambda_{ij} \, P^{-(j+1)},$$

wo λ_{ij} ganz rational ist und $0 \leqq \lambda_{ij} \leqq P-1$. Es gibt einen Index $n_i \geqq 0$ derart, dasz bei Entwicklung jeder Zahl von I_i in eine Potenzreihe nach P^{-1} die ersten n_i Koeffizienten gleich λ_{ij} sind ($j = 0, 1, \ldots, n_i - 1$). Aber im allgemeinen liegt nicht jede Zahl deren ersten n_i Koeffizienten gleich λ_{ij} sind in I_i: um das einzusehen genügt es a_{i+1} irrational zu nehmen. Ein ein-eindeutiger Verband zwischen n_i und dem Intervalle kann man erhalten wenn man erstens für die Randpunkte der I_i rationale Zahlen nimmt. Da das Masz der Menge aller rationalen Zahlen Null ist und sie überall dicht liegen, kann dies in derartiger Weise geschehen, dasz das Masz der neuen Menge O' das Masz $m_L O$ von O höchstens um eine willkürlich gegebene Zahl $\delta > 0$ übertrifft. In der Definition des äuszeren LEBESGUEschen Maszes einer willkürlichen Menge als untere Grenze der Maszen aller die gegebene Menge umfassenden offenen Mengen, können wir also ohne Beschwerde voraussetzen, dasz die offenen Mengen der genannten Bedingung genügen. Wir unterlassen daher die Striche. Man teile dann jeder I_i in eine endliche Anzahl Teilintervallen derart, dasz die Länge jeder dieser neuen Intervallen gleich P^{-k} ist für passendes k. Schreibt man dann in der Entwicklung des rechten Randpunktes für den letzten Koeffizienten x ein $x-1$, gefolgt durch unendlich viele $P-1$, dann ist erreicht, dasz jeder der jetzt wieder I_i zu nennenden Intervallen die und nur die Zahlen umfaszt deren Entwicklung in einer Potenzreihe nach P^{-1} einen gegebenen Anfangsabschnitt haben. Ist n_i der korrespondierende Index, so sind P^{n_i} Kombinationen für die ersten n_i Koeffizienten möglich und damit korrespondieren in $<0, 1>$ P^{n_i} gleiche Intervallen mit Länge P^{-n_i} Also $\varepsilon_i = P^{-n_i}$ und

$$m_L \, O = \sum_{i=1}^{\infty} P^{-n_i} .$$

Der Verband mit dem P-adischen Masze ist jetzt wie folgt zu legen. Es sei $K_t(P)$ die

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[1]) H. TURKSTRA, Metrische bijdragen tot de theorie der diophantische approximaties in het lichaam der P-adische getallen. Diss. V. U. Amsterdam 1938.

Menge der P-adischen Zahlen deren Wert $\leqq P^{-t}$ ist (t ganz rational). Jede dieser Zahlen kann geschrieben werden in der Form einer P-adisch konvergenten Potenzreihe in P:

$$x = \sum_{j=0}^{\infty} x_j \, P^{j+t},$$

mit x_j ganz rational und $0 \leqq x_j \leqq P—1$. Zwischen die vorher genannten Menge R und $K_t\,(P)$ besteht also eine ein-eindeutige Korrespondenz. Dem Intervalle I_i ist die Menge aller Zahlen aus $K_t\,(P)$ mit gegebenen Anfangsabschnitt in der Potenzreihenentwicklung zugefügt; sei diese Menge E_{n_i} TURKSTRA nennt E_{n_i} eine n_i —*te* „takverzameling". E_{n_i} ist also das Analogon des Intervalles im Körper der reellen Zahlen. TURKSTRA stellt das Masz auf P^{-n_i} das heiszt gleich dem Masze des Bildes I_i. Ist $m_p\overline{O}$ das Masz des Bildes in $K_t\,(P)$ von O in R, so ist

$$m_p\,\overline{O} = m_L\,O.$$

Die Masztheorie in R läszt sich also unmittelbar auf $K_t\,(P)$ übertragen indem man eine Menge und das Bild das gleiche Masz zuordnet. Der Uebergang zu nicht beschränkten Menge in $K(P)$ läszt sich dann ausführen wie bei TURKSTRA.

2. Mit dem LEBESGUEschen Masz in R korrespondiert als Abstand zweier Punkte in R der absolute Wert der Differenz der diesen Punkte gehörenden reellen Zahlen.

Ebenso definiert man der Abstand zweier Punkte aus $K_t\,(P)$ als der P-adische Wert der Differenz der diesen Punkte zugeordneten P-adischen Zahlen.

Der Begriff der Umgebung ist damit festgelegt in R und $K_t\,(P)$. Die obenbenutzte Transformation von R in $K_t(P)$ ist aber bei diesen Festsetzungen nicht stetig. Man sieht das an einem Beispiel. Es sei a die rationale Zahl $a = 0, \lambda_0\lambda_1 \ldots \lambda_i$ mit $0 \leqq \lambda_k \leqq P—1$. Die reellen Zahlen zwischen $0, \lambda_0 \ldots \lambda_i 1$ und a und die zwischen a und $0, \lambda_0 \ldots \overline{\lambda_i—1}\;\overline{P—1}$ liegen alle in derselben Umgebung von a mit Länge $2\,P^{-(i+2)}$ d.h. für diese Zahlen x gilt $|x—a| \leqq P^{-(i+2)}$ und das so bestimmte Intervall enthält nur diese Zahlen. Die korrespondierende P-adische Zahlen sind

$$\begin{aligned} \alpha' &= \sum_{x=0}^{i-1} \lambda_x \, P^{x+t} + (\lambda_i - 1)\, P^{i+t} + (P-1)\, P^{i+t+1}, \\ \alpha &= \sum_{x=0}^{i} \lambda_x \, P^{x+t}, \\ \alpha'' &= \sum_{x=0}^{i} \lambda_x \, P^{k+t} + P^{i+t+1}. \end{aligned}$$

Der Abstand von α und α' ist definitionsgemäsz $P^{-(i+t)}$; der Abstand von α und α'' aber $P^{-(i+t+1)}$. Die korrespondierende P-adische Zahlen bilden also nicht wieder eine volle Umgebung von α. Es liegt dies daran, dasz beim P-adischen Abstand der Index des ersten nicht übereinstimmenden Koeffizientes in der Entwicklung der zwei Zahlen maszgebend, während dies nicht der Fall ist beim gewöhnlichen Abstand im Körper der reellen Zahlen.

Es ist diese Diskontinuität der Abbildung die Ursache dasz das Intervall I_i abgeschlossen war (und auch die Menge aller reellen Zahlen mit gegebenen Anfangsabschnitte) während das Bild E_{n_i} bekanntlich sowohl offen als abgeschlossen ist 1)).

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1) Die Menge der irrationalen Zahlen mit vorgegebenen Anfangsabschnitt in der Reihenentwicklung nach absteigenden Potenzen von P bildet in bezug auf die Menge I aller irrationalen Zahlen eine offene und geschlossene Menge, wie man leicht einsieht. Allerdings besitzt in I nicht jede begrenzte Menge einen Häufungspunkt.

Bemerkt sei noch dasz aus dem obigen deutlich hervorgeht dasz die Menge aller P-adischen Zahlen mit gegebenem Anfangsabschnitt die naturgemäsze Verallgemeinerung des Begriffes „Intervall" ist.

3. Die beiden Masztheorien können auf folgende Weise unter einen Gesichtspunkt gebracht werden.

Es sei F die Menge aller Folgen ganzer rationaler Zahlen mod. P. Die Elementen kann man auffassen als Punkte eines Raumes mit unendlich vielen Dimensionen. Dieser Raum kann ein-eindeutig abgebildet werden sowohl auf R wie auf $K_t(P)$. Der Raum ist metrisch: als Abstand zweier Punkten kann man nehmen den Abstand der Bildpunkten entweder in R, entweder in $K_t(P)$. In beiden Fällen erfüllt die Metrik die zu stellenden Forderungen. Das durch die Abbildung auf R bzw. $K_t(P)$ in F induzierte Masz einer Menge ist aber in beiden Fällen dasselbe. Es geht dann deutlich der Verband zwischen LEBESGUEschem Masz und P-adischem Masz hervor.

Bemerken wir noch, dasz der Raum F ein BAIREscher Nullraum ist worin man zwei Punkten $(\lambda_0, \lambda_1, \ldots)$ und $(\mu_0, \mu_1, \ldots)$ als nicht voneinander verschieden ansieht falls $\lambda_i \equiv \mu_i$ mod. P $(i = 0, 1, \ldots)$.

4. Die Anleitung zu obigen Bemerkungen war die Frage, wasz im Körper der P-adischen Zahlen werde aus dem bekannten Theorem BORELs bezüglich die Verteilung der Koeffizienten der Reihenentwicklung fast aller reellen Zahlen nach absteigenden Potenzen von P (die BORELschen „normale" Zahlen)[1]. Man findet die folgende Eigenschaft:

Es sei $\alpha_n = \frac{f_n}{n}$ *die Frequenz der Zahl* a $(0 \leqq a \leqq P-1)$ *in den ersten* n*-Koeffizienten der Reihenentwicklung der* P*-adischen Zahl* x *nach steigenden Potenzen von* P*. Dann hat die Menge aller* P*-adischen Zahlen deren Folge* $\{\alpha_n\}$ $(n = 1, 2, \ldots)$ *einen Häufungspunkt ungleich* $\frac{1}{P}$ *hat, das* P*-adische Masz Null.*

Der Beweis verläuft ganz wie bei BOREL. Es handelt sich dabei nur um die Interpretation einer Wahrscheinlichkeit in einer Masztheorie.

Den Haag, September 1942.

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[1] E. BOREL. Traité du calcul des probabilités et de ses applications t. II fasc. 1. Applications à l'arithmétique. Paris 1926.

Mathematics. — *Zur Geometrie der P-adischen Zahlen.* Von A. F. MONNA. (Communicated by Prof. W. VAN DER WOUDE.)

(Communicated at the meeting of November 28, 1942.)

§ 1. In dieser Zeitschrift hat C. VISSER einen neuen Beweis eines Satzes von BLICHFELDT–JESSEN aus der Geometrie der Zahlen und eine Verallgemeinerung dieses Satzes gegeben ¹). Wir werden zeigen, dasz die Sätze fast genau so ausgesprochen werden können wenn man den Betrachtungen anstatt des Körpers der reellen Zahlen den Körper $K(P)$ der P-adischen Zahlen zugrunde legt. Die Möglichkeit dazu beruht im wesentlichen auf der von TURKSTRA gegebenen Masztheorie in $K(P)$ ²). Der nicht-archimedische Charakter von $K(P)$ macht einige Aenderungen in den Beweisen notwendig.

§ 2. Es sei $K_n(P)$ der Produktraum $\{K(P)\}^n$, d.h. der Raum dessen Punkte die n-Tüpel $(x_1, x_2, \ldots, x_n)$ sind wo x_i ein Element von $K(P)$ ist; es ist $K_1(P) \equiv K(P)$.

$K(P)$ wird bekanntlich zu einem metrischen topologischen Raum wenn man als Abstand zweier Punkte x und y aus $K(P)$ definiert $r_{xy} = |x-y|_P$, wo $|\alpha|_P$ die P-adische Bewertung des Elementes α bedeutet. Man hat

$$|\alpha+\beta|_P \leqq \max\left[|\alpha|_P, |\beta|_P\right]. \qquad (1)$$

Der Raum $K_n(P)$ ist ebenfalls metrisch und topologisch: der Abstand, mit desser Hilfe der Begriff Umgebung festgelegt wird, kann dabei noch auf verschiedene Weisen definiert werden. Als Abstand der Punkte $(x_1, \ldots, x_n) \equiv x$ und $(y_1, \ldots, y_n) \equiv y$ kann man z.B. nehmen

$$r_{xy} = \max_{i=1,\ldots,n} \left[|x_i - y_i|_P\right], \qquad (2)$$

oder

$$r_{xy} = \left\{ \sum_i |x_i - y_i|_P^p \right\}^{\frac{1}{p}} \qquad (p \geqq 1). \qquad (3)$$

Im ersten Fall gilt als Verallgemeinerung von (1)

$$r_{x,-y} \leqq \max\left[r_{x,0}, r_{y,0}\right] \qquad (4)$$

und

$$r_{x,-y} = \max\left[r_{x,0}, r_{y,0}\right] \text{ falls } r_{x,0} \neq r_{y,0}. \qquad (4a)$$

Man hat nämlich

$$r_{x,-y} = \max_i \left[|x_i + y_i|_P\right] \leqq \max_i \left[\max\left[|x_i|_P, |y_i|_P\right]\right] =$$

$$= \max\left[\max_i \left[|x_i|_P\right], \max_i \left[|y_i|_P\right]\right] = \max\left[r_{x,0}, r_{y,0}\right].$$

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¹) C. VISSER, On a theorem in the geometry of numbers and a property of mass distributions in n-dimensional space. Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **42**, 487—490 (1939).

²) H. TURKSTRA, Metrische bijdragen tot de theorie der diophantische approximaties in het lichaam der P-adische getallen. Diss. V. U. Amsterdam 1936.

Zum Beweise von (4*a*) kann man annehmen $r_{x,0} > r_{y,0}$ oder $\max\limits_i (|x_i|_P) > \max\limits_i (|y_i|_P)$ und weiter ohne Beschränkung der Allgemeinheit $\max (|x_i|_P) = |x_1|_P$. Dann folgt $|x_1|_P > |y_i|_P$ $(i=1, \ldots n)$. Wegen $|x_1+y_1|_P = |x_1|_P$ und $|x_k+y_k|_P \leqq \max[|x_k|_P, |y_k|_P] < < |x_1|_P$ folgt dann $\max\limits_i [|x_i+y_i|_P] = |x_1|_P$ d.h. (4a).

Im Falle (3) gelten aber (4) und (4*a*) nicht allgemein, wie man folgendermaszen einsieht. Es seien die x_i und y_i rationale Zahlen und

$$x_i = P^{f_i} \frac{a_i}{b_i}, \; y_i = P^{g_i} \frac{c_i}{d_i} \qquad (i = 1, 2, \ldots n)$$

wo a_i, b_i, c_i, d_i, f_i, g_i ganze rationale Zahlen sind und a_i, b_i, c_i, d_i, P relativ prim. Wir können annehmen dasz $\Sigma |x_i|_P^p > \Sigma |y_i|_P^p$ d h. $\Sigma P^{-pf_i} > \Sigma P^{-pg_i}$. Die x_i und y_i können derart gewählt werden dasz $f_1 > g_1$ und $f_j < g_j$ $(j = 2, \ldots, n)$, also

$$|x_1|_P < |y_1|_P, \; |x_j|_P > |y_j|_P.$$

Dann ist

$$\sum_{=1}^{n} |x_i + y_i|_P^p = P^{-pg_1} + \sum_{i=2}^{n} P^{-pf_i} > \sum_{i=1}^{n} P^{-pf_i} =$$

$$= \sum_{i=1}^{n} |x_i|_P^p = \max [\Sigma |x_i|_P^p, \Sigma |y_i|_P^p],$$

die Ungleichung (4) ist also nicht erfüllt. Man sieht, dasz in diesem Fall (4) nur gilt wenn $|x_i|_P > |y_i|_P$ für $i = 1, 2, \ldots, n$.

Bei der Definition (3) gilt also nur die gewöhnliche Dreiecksungleichung und nicht die verschärfte.

Wir bemerken noch, dasz $K_n(P)$ ein hyperkomplexes System ist mit Einheiten $e_1 = (1, 0, \ldots)$, $e_2 = (0, 1, 0, \ldots)$ u.s.w. falls die Multiplikation der e_i untereinander in übliche Weise definiert wird. Es ist dann $x = \Sigma x_i e_i$. Dieses System wird im allgemeinen Nullteiler besitzen und daher ist $K_n(P)$ für $n > 1$ im allgemeinen kein Körper [1]). Daher ist im Vorangehenden zwar der Abstandsbegriff in $K_n(P)$ definiert aber nicht die Bewertung von $K_1(P)$ zu $K_n(P)$ erweitert. Es ist deutlich, dasz falls $K_n(P)$ Nullteiler hat und daher kein Körper ist, der jeder Bewertung zu stellenden Forderung $\varphi(ab) = = \varphi(a)\,\varphi(b)$ nicht allgemein genüge geleistet werden kann.

Ist aber $K_n(P)$ ein Körper, so kann in K_n auf eine und nur eine Weise eine Bewertung derart definiert werden, dasz sie in $K_1(P)$ mit der P-adischen Bewertung übereinstimmt. K_n ist dann nämlich eine algebraische Erweiterung von K_1, da K_n endlich ist. Die Bewertung des Elementes x ist dann

$$|x|_P = \sqrt[n]{|N_{k_n(P)}(x)|_P} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

wo N_{k_n} die Norm von x in $K_n(P)$ bedeutet [2]). Diese Bewertung ist nicht-archimedisch und daher gelten für den Abstand $r_{x,y} = |x-y|_P$ die Beziehungen (4) und (4*a*).

Für $n = 2$ und Einheiten 1 und i mit $i^2 = -1$, hat K_n keine Nullteiler. Aus $(a + ib)$

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[1]) Man vergleiche das FROBENIUSsche Theorem, welches ausdrückt dasz die einzigen hyperkomplexen Systeme ohne Nullteiler über den Körper der reellen Zahlen sind: *a*) der Körper der reellen Zahlen, *b*) der Körper der komplexen Zahlen und *c*) der Körper der reellen Quaternionen (einen einfachen Beweis gibt L. E. DICKSON in „Linear algebras") (Cambridge 1914, S. 10).

[2]) Sehe VAN DER WAERDEN, Moderne Algebra I (Berlin 1937) S. 262.

$(c + id) = 0$ (wo a, b, c und d P-adische Zahlen sind) folgt nämlich $ac - bd = 0$ und $ad + bc = 0$, also, wenn $c \neq 0$, $d \neq 0$, auch $\frac{a}{d} = \frac{b}{c}$. Multiplikation gibt dann $a^2 = -b^2$. Da a und b P-adischer Limes rationaler Zahlen a_n und b_n sind, gilt also für $m > N(\varepsilon)$: $|a_m^2 + b_m^2|_P < \varepsilon$, also $|a_m|_P$ und $|b_m|_P < \sqrt{\varepsilon}$ d.h. $a = b = 0$. Man findet in diesem Fall aus (5)

$$|x|_P = \sqrt{|x_1^2 + x_2^2|_P} \quad \text{1)}.$$

Die im Folgenden zu beweisenden Sätze sind unabhängig von einer allgemeinen Metrik in $K_n(P)$. Es wird der Begriff des Abstandes nur für Punkte benutzt, die in genau einer der Koordinaten von einander verschieden sind; hat z.B. diese Koordinate den Index i, so setzen wir den Abstand $|x_i - y_i|_P$.

Die Masztheorie in $K(P)$ läszt sich durch Bildung des Produktmaszes in übliche Weise zu $K_n(P)$ erweitern.

Unter dem n-dimensionalen Intervall H_N^a mit Mittelpunkt $a \equiv (a_1, \ldots, a_n)$ sei verstanden die Menge der Punkte x für welche

$$|a_i - x_i|_P \leqq P^{-N}. \qquad (6)$$

$(i = 1, \ldots, n;$ N ganz rational).

Das Masz von H_N^a ist gleich $(P^{-N})^n$.

Für eine Punktreihe $\xi^{(1)}, \xi^{(2)}, \ldots$ in $K_n(P)$ sei die obere Dichte definiert durch

$$\limsup_{\substack{a \text{ willkürlich} \\ P^{-N} \to \infty}} \frac{\text{Anzahl der } \xi^{(i)} \text{ in } H_N^a}{(P^{-N})^n}. \qquad (7)$$

Bei dieser Definition ist zu bemerken, dasz es in $K_n(P)$ in der Tat Folgen von Intervallen gibt deren Masz gegen unendlich strebt. Man hat nämlich

$$x_i = \sum_{j=-N}^{\infty} x_{ij} P^j$$

mit $|x_i|_P = P^N$. Man braucht also nur die, immer endliche, Anzahl der Summanden mit negativen Potenzen von P unbestimmt wachsen zu lassen.

Unter $E + y$, wo E eine Menge in $K_n(P)$ und y ein Punkt ist, sei mit VISSER (l.c.) die Menge der Punkte $x + y$ mit x in E verstanden.

§ 4. *Lemma. Es sei* $F_1, F_2, \ldots$ *eine endliche oder abzählbar unendliche Folge meszbarer Teilmengen einer gegebenen Menge* S *mit endlichem Masz* > 0 *in* K_n. *Es gibt dann einen Punkt derart, dasz die Anzahl der Mengen* F_i, *wozu er gehört* $\geq \frac{\Sigma m(F_i)}{m(S)}$ *ist.*

Beim Beweise des analogen Lemmas im Reellen benutzt VISSER die LEBESGUE-Integration. Da die Bewertung in $K(P)$ diskret ist, kann die Definition des LEBESGUEschen Integrals einer Funktion nicht unmittelbar auf P-adische Funktionen übertragen werden. Eine Definition des Integrals der beim Beweise zu benutzenden Funktionen musz deshalb vorausgeschickt werden.

Definition. Das Gebiet S *worauf die* P*-adische Funktion* $f(x)$ *definiert ist sei die*

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1) Setzt man allgemeiner

$$r_{x,y} = \sqrt{|\Sigma (x_i - y_i)^2|_P},$$

so beweist man leicht die Gültigkeit von (4) und (4a), wenn man beachtet dasz x_i und y_i P-adische Limites rationaler Zahlen sind.

Summe einer endlichen oder abzählbar unendlichen Anzahl meszbarer Mengen E_i *mit Masz* $m(E_i)$, *derart dasz*

$$f(x) = C_i \text{ auf } E_i \quad (C_i \textit{ konstante P-adische Zahl}).$$

Dann sei das Integral von $f(x)$ *über* S *definiert durch*

$$\int\limits_S f(x)\, dx_P = \sum_i C_i\, m(E_i). \quad \text{. . . . .} \quad (8)$$

Man zeigt leicht dasz, falls $f_k(x)$ $(k = 1, 2, \ldots)$ derartige Treppenfunktionen sind,

$$\sum_k \int\limits_S f_k(x)\, dx_P = \int\limits_S \sum_k f_k(x)\, dx_P \quad \text{. . . . . . .} \quad (9)$$

falls eins der beiden Glieder existiert.

Nimmt $f(x)$ auszerdem nur rationale Werte an, so folgt aus $f(x) < C$ auf S (C rational)

$$\int\limits_S f(x)\, dx_P < C\, m(S). \quad \text{. . . . .} \quad (10)$$

Setzt man in (9) für f_k die karakteristische Funktion von F_k ein, so folgt mit Hilfe der Ungleichung (10) wie bei VISSER das Lemma (es ist die karakteristische Funktion in der Tat eine P-adische Treppenfunktion).

§ 5. *Satz 1. Es seien* $\xi^{(i)}$ $(i = 1, 2, \ldots)$ *eine Punktfolge mit oberer Dichte* $> \lambda$ *und* E *eine meszbare Menge mit endlichem Masz* $m(E)$. *Dann gibt es einen Punkt* y *derart, dasz zu* $E + y$ *mehr als* $\lambda m(E)$ *der Punkte* $\xi^{(i)}$ *gehören.*

Da einige Aenderungen im Beweise erförderlich sind, lassen wir ihn ganz folgen. Wir setzen voraus, dasz E im Intervall H_d^0 liegt (wo $0 \equiv (0, 0, \ldots)$) d.h.

$$|x_i|_P \leqq P^{-d} \; (i = 1, \ldots, n) \quad \text{. . . . .} \quad (11)$$

für alle x in E. Es können a und N derart gewählt werden ($-N$ genügend grosz) dasz

$$\frac{\text{Anzahl der } \xi^{(i)} \text{ in } H_N^a}{(P^{-N})^n} > \lambda. \quad \text{. . . . .} \quad (12)$$

Man wählte auszerdem $-N$ so grosz dasz

$$N \leqq d. \quad \text{. . . . .} \quad (13)$$

Man hat

$$\sum_i m(E + a \,.\, H_N^{\xi^{(i)}}) \geqq \sum_{\xi^{(i)} \text{ in } H_N^a} m(E + a \,.\, H_N^{\xi^{(i)}}). \quad \text{. . . .} \quad (14)$$

Es gilt

$$E + a \subset H_N^{\xi^{(i)}} \text{ falls } \xi^{(i)} \text{ in } H_N^a \text{ liegt.} \quad \text{. . . . . .} \quad (15)$$

Es sei um dies zu zeigen, x einen Punkt von $E + a$. Da E in H_d^0 liegt, gehört x dann zu H_d^a [1]), also

$$|x_k - a_k| \leqq P^{-d} \; (k = 1, \ldots, n). \quad \text{. . . . . . .} \quad (16)$$

---

[1]) In der Tat, ist α ein Punkt von E, d.h. $|\alpha_i|_P \leq P^{-d}$, so ist

$$|(\alpha_i + a_i) - a_i|_P \leqq P^{-d}$$

also liegt $\alpha + a$ in H_d^a.

$\xi^{(i)}$ in H_N^a ergibt

$$| \xi_k^{(i)} - a_k | \leqq P^{-N} (k = 1, \dots, n). \quad . \; . \; . \; . \; . \; . \quad (17)$$

Nun ist

$$| \xi_k^{(i)} - x_k |_P = | \xi_k^{(i)} - a_k + (a_k - x_k) |_P \leqq \max \left[| \xi_k^{(i)} - a_k |_P, | a_k - x_k |_P \right]. \quad (18)$$

Es genügt den Fall zu betrachten dasz $| \xi_k^{(i)} - a_k |_P = P^{-N}$. Wegen (13), (16) und (17) ist dann das Maximum in (18) gleich P^{-N}, also

$$| \xi_k^{(i)} - x_k |_P \leqq P^{-N} (k = 1, \dots, n),$$

d.h. der Punkt x liegt in $H_k^{\xi^{(i)}}$ infolge der Definition. Damit ist (15) gezeigt.

Wegen der Invarianz des Maszes bei einer Translation folgt jetzt aus (14) und (15) $\sum_i m (E + a \,.\, H_N^{\xi^{(i)}} \geqq \sum_{\xi^{(i)} \text{ in } H_N^a} m (E + a) = m (E)$. Anzahl der $\xi^{(i)}$ in $H_N^a >$

$$> \lambda m (E) \,.\, (P^{-N})^n.$$

Es sei

$$F_i = E + a \,.\, H_N^{\xi^{(i)}} - \xi^{(i)}.$$

Man zeigt wie bei (15)

$$F_i \subset H_N^0.$$

Also

$$\sum m (F_i) = \sum_i m (E + a \,.\, H_N^{\xi^{(i)}}) > \lambda m (E) (P^{-N})^n.$$

Wegen des Lemmas gibt es dann einen Punkt x dasz zu mehr als $\dfrac{\lambda m (E) (P^{-N})^n}{m (H_N^a)} = \lambda m(E)$ der F_i's gehört, sagen wir zu

$$F_{i_1}, \dots, F_{i_p}.$$

Also

$$x \in E + a \,.\, H_N^{\xi^{(i_k)}} - \xi^{(i_k)},$$

$$x + \xi^{(i_k)} \in E + a \,.\, H_N^{\xi^{(i_k)}}.$$

A fortiori

$$x + \xi^{(i_k)} \in E + a$$

oder

$$\xi^{(i_k)} \in E + a - x \qquad (k = 1, \dots, p; p > \lambda m (E).$$

$a - x$ ist also der gesuchte Punkt y.

§ 6. Es sei $\sigma(E)$ eine nicht negative total additive Mengenfunktion, definiert auf einen Mengenkörper in $K_n(P)$ dasz die BOREL'schen Mengen umfaszt. Damit der zweite Satz von VISSER auf den Raum $K_n(P)$ übertragen werden kann, ist es erförderlich $\sigma(E)$ eine Beschränkung auf zu erlegen. Wir setzen voraus, dasz $\sigma(E)$ absolut stetig ist in Bezug auf das Produktmasz $m(E)$ in $K_n(P)$ d.h. dasz $\sigma(E) \to 0$ falls $m(E) \to 0$ ($\sigma(E)$ ist reellwertig).

Die obere Dichte von $\sigma(E)$ sei definiert durch

$$\limsup_{\substack{a \text{ willkürlich} \\ P^{-N} \to \infty}} \frac{\sigma (H_N^a)}{(P^{-N})^n}.$$

Satz 2. Es seien σ eine absolut stetige Mengenfunktion mit obere Dichte $> \lambda$ und E eine willkürliche Menge mit endlichem inneres Masz $m_i(E)$. Dann gibt es einen Punkt y derart dasz $\sigma(E+y) > \lambda m_i(E)$.

Wir betrachten $m(E + a \,.\, H_N^{\xi})$ als Funktion von ξ. In einem Cartesischen Raum ist sie eine stetige Funktion von ξ und demzufolge meszbar i.b.a. σ. Diese Stetigkeit gilt im allgemeinen nicht in $K_n(P)$.

Gehen wir von einem Punkte ξ_0 aus. Liegt ξ in $H_N^{\xi_0}$, so ist $H_N^{\xi_0} = H_N^{\xi}$. Daraus folgt, dasz $m(E + a \,.\, H_N^{\xi_0}) = m(E + a \,.\, H_N^{\xi})$ wenn ξ in $H_N^{\xi_0}$ liegt, also ist die zu betrachtende Funktion konstant in $H_N^{\xi_0}$. Bemerkt sei dasz $H_N^{\xi_0}$ abgeschlossen ist. Liegt ξ nicht in $H_N^{\xi_0}$ so haben H_N^{ξ} und $H_N^{\xi_0}$ keinen Punkt gemein [1]). Es folgt daraus dasz $m(E + a \,.\, H_N^{\xi})$ Diskontinuitäten haben kann wenn ξ den Rand von $H_N^{\xi_0}$ passiert. Geht man von einem anderen Punkte ξ_0 aus, so findet man offenbar dasselbe Resultat. Diese Eigentümlichkeit bringt mit sich, dasz wir, damit diese Funktion von ξ meszbar i.b.a. σ werde, voraussetzen müssen dasz $\sigma(E) = 0$ falls $m(E) = 0$. Die eventuelle Diskontinuitätspunkte liegen dann in einer Menge wofür $\sigma = 0$ ist.

Unter Beachtung der beim Beweise des Satzes 1 eingeführten Abänderungen kann der Beweis jetzt vollendet werden wie bei VISSER. Bemerkt sei, dasz die Integrationen, die in Bezug auf die reellwertige Mengenfunktion σ stattfinden, hier keine Schwierigkeit bieten wie beim Beweise des Lemmas.

§ 7. Eine weitergehende Generalisation der Sätze ist möglich. Man sieht ein, dasz die Sätze ausgesprochen werden können für jeden Banachraum in dem sich eine total additive reellwertige Mengenfunktion definieren läszt die nicht identisch Null ist. Zu einem HILBERTraum können die Sätze nicht unmittelbar erweitert werden: ein derartiges Masz gibt es darin nicht.

Für die Erweiterung zu einem Vektorraum ist übrigens notwendig, dasz der Raum normiert ist. Es ist ja für die Sätze wesentlich dasz der Raum Translationen gestattet, d.h. sind z_1 und z_2 zwei Punkte und ist $\varrho(z_1, z_2)$ ihr Abstand, so soll für jedes a gelten

$$\varrho(z_1 + a, z_2 + a) = \varrho(z_1, z_2).$$

Also auch $(z_2 - z_1, 0) = \varrho(z_2, z_1)$ und ϱ ist die gesuchte Norm.

Obgleich für den BAIREschen Nullraum sich eine Masztheorie entwicklen läszt, gelten jedoch die Sätze nicht bei der gewöhnlichen Definition des Abstandes in diesem Raum. Den Abstand setzt man nämlich gleich $\frac{1}{n}$ wenn sich die beiden Punkte (Folgen) im oberen Index des n-ten Symbols unterscheiden, aber in keinem früheren. Es gibt hier also keine Norm.

Den Haag, Oktober 1942.

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[1]) Man hat nämlich die Eigenschaft dasz zwei Intervallen mit demselben N entweder identisch sind oder ganz auszerhalb einander liegen (Sehe TURKSTRA l.c. S. 79).

Mathematics. — *Extension d'une série des fonctions de* BESSEL, *due à* LOMMEL, *et de quelques séries des fonctions de* BESSEL *analogues.* II. By J. G. RUTGERS. (Communicated by Prof. J. A. SCHOUTEN.)

(Communicated at the meeting of November 28, 1942.)

Pour $k=0$, $k=1$ et $k=2$ on obtient de *Va, VIa, Vb* et *VIb* les formules particulières:

$$\sum_{n=0}^{\infty}(-1)^n\left(2n+\tfrac{1}{2}\right)I_{2n+\frac{1}{2}}(x)=\sqrt{\frac{x}{2\pi}}\cos x, \quad \ldots \quad (12)$$

$$\sum_{n=0}^{\infty}(-1)^n\left(2n+\tfrac{3}{2}\right)I_{2n+\frac{3}{2}}(x)=\sqrt{\frac{x}{2\pi}}\sin x, \quad \ldots \quad (13)$$

$$\sum_{n=0}^{\infty}(-1)^n\left(2n+\tfrac{1}{2}\right)^3 I_{2n+\frac{1}{2}}(x)=\sqrt{\frac{x}{2\pi}}\left(\tfrac{1}{4}\cos x-2x\sin x\right), \quad . \quad (14)$$

$$\sum_{n=0}^{\infty}(-1)^n\left(2n+\tfrac{3}{2}\right)^3 I_{2n+\frac{3}{2}}(x)=\sqrt{\frac{x}{2\pi}}\left(\tfrac{1}{4}\sin x+2x\cos x\right), \quad . \quad (15)$$

$$\sum_{n=0}^{\infty}(-1)^n\left(2n+\tfrac{1}{2}\right)^5 I_{2n+\frac{1}{2}}(x)=\sqrt{\frac{x}{2\pi}}\left\{\tfrac{1}{16}\left(1-128x^2\right)\cos x-5x\sin x\right\}, . \quad (16)$$

$$\sum_{n=0}^{\infty}(-1)^n\left(2n+\tfrac{3}{2}\right)^5 I_{2n+\frac{3}{2}}(x)=\sqrt{\frac{x}{2\pi}}\left\{\tfrac{1}{16}\left(1-128x^2\right)\sin x+5x\cos x\right\}, . \quad (17)$$

$$\left.\begin{aligned}&\sum_{n=0}^{\infty}(-1)^n\left(2n+\tfrac{1}{2}\right)^7 I_{2n+\frac{1}{2}}(x)=\\&\qquad=\sqrt{\frac{x}{2\pi}}\left\{\tfrac{1}{64}\left(1-4480x^2\right)\cos x-\tfrac{1}{8}x\left(91-384x^2\right)\sin x\right\}, .\end{aligned}\right\}(18)$$

$$\left.\begin{aligned}&\sum_{n=0}^{\infty}(-1)^n\left(2n+\tfrac{3}{2}\right)^7 I_{2n+\frac{3}{2}}(x)=\\&\qquad=\sqrt{\frac{x}{2\pi}}\left\{\tfrac{1}{64}\left(1-4480x^2\right)\sin x+\tfrac{1}{8}x\left(91-384x^2\right)\cos x\right\}, .\end{aligned}\right\}(19)$$

$$\left.\begin{aligned}&\sum_{n=0}^{\infty}(-1)^n\left(2n+\tfrac{1}{2}\right)^9 I_{2n+\frac{1}{2}}(x)=\\&=\sqrt{\frac{x}{2\pi}}\left\{\tfrac{1}{256}\left(1-123648x^2+98304x^4\right)\cos x-\tfrac{1}{8}x\left(205-8064x^2\right)\sin x\right\},\end{aligned}\right\}(20)$$

$$\left.\begin{aligned}&\sum_{n=0}^{\infty}(-1)^n\left(2n+\tfrac{3}{2}\right)^9 I_{2n+\frac{3}{2}}(x)=\\&=\sqrt{\frac{x}{2\pi}}\left\{\tfrac{1}{256}\left(1-123648x^2+98304x^4\right)\sin x+\tfrac{1}{8}x\left(205-8064x^2\right)\cos x\right\},\end{aligned}\right\}(21)$$

$$\left.\begin{aligned}&\sum_{n=0}^{\infty}(-1)^n\,(2n+\tfrac{1}{2})^{11}\,I_{2n+\frac{1}{2}}(x)=\\&=\sqrt{\frac{x}{2\pi}}\,\Big\{\tfrac{1}{1024}\,(1-3196160\,x^2+16220160\,x^4)\cos x-\\&\qquad-\tfrac{1}{128}\,x\,(7381-1951488\,x^2+491520\,x^4)\sin x\Big\},\end{aligned}\right\}\quad(22)$$

$$\left.\begin{aligned}&\sum_{n=0}^{\infty}(-1)^n\,(2n+\tfrac{3}{2})^{11}\,I_{2n+\frac{3}{2}}(x)=\\&=\sqrt{\frac{x}{2\pi}}\,\Big\{\tfrac{1}{1024}\,(1-3196160\,x^2+16220160\,x^4)\sin x+\\&\qquad+\tfrac{1}{128}\,x\,(7381-1951488\,x^2+491520\,x^4)\cos x\Big\},\end{aligned}\right\}\quad(23)$$

5. Examinons maintenant pour ν arbitraire et k entier $\geqq 0$ la série:

$$S_{\nu,2k+2}(x)=\sum_{n=0}^{\infty}(-1)^n\,(\nu+2n)^{2k+2}\,I_{\nu+2n}(x)\quad.\;.\;.\;.\;.\quad(24)$$

En appliquant des réductions analogues à celles des §§ 1, 2 et 3, nous trouvons successivement les formules:

$$\begin{aligned}S_{\nu,2k+2}(x)&=\frac{x}{2}\sum_{n=0}^{\infty}(-1)^n\,(\nu+2n)^{2k+1}\,\{I_{\nu+2n-1}(x)+I_{\nu+2n+1}(x)\}=\\&=\nu^{2k+1}\,\frac{x}{2}\,I_{\nu-1}(x)-\frac{x}{2}\sum_{n=0}^{\infty}(-1)^n\,\{(\nu+2n+2)^{2k+1}-(\nu+2n)^{2k+1}\}\,I_{\nu+2n+1}(x)=\\&=\nu^{2k+1}\,\frac{x}{2}\,I_{\nu-1}(x)-\frac{x}{2}\sum_{n=0}^{\infty}(-1)^n\,I_{\nu+2n+1}(x)\sum_{p=0}^{2k+1}\binom{2k+1}{p}(\nu+2n+1)^p\{1+(-1)^p\}=\\&=\nu^{2k+1}\,\frac{x}{2}\,I_{\nu-1}(x)-x\sum_{p_1=0}^{k}\binom{2k+1}{2p_1}S_{\nu+1,2p_1}(x),\end{aligned}$$

resp.

$$\begin{aligned}S_{\nu,2k+2}(x)&=(\nu-2)^{2k+1}\,\frac{x}{2}\,I_{\nu-1}(x)+\frac{x}{2}\sum_{n=0}^{\infty}(-1)^n\,\{(\nu+2n)^{2k+1}-(\nu+2n-2)^{2k+1}\}\,I_{\nu+2n-1}(x)=\\&=(\nu-2)^{2k+1}\,\frac{x}{2}\,I_{\nu-1}(x)+\frac{x}{2}\sum_{n=0}^{\infty}(-1)^n\,I_{\nu+2n-1}(x)\sum_{p=0}^{2k+1}\binom{2k+1}{p}(\nu+2n-1)^p\,\{1+(-1)^p\}=\\&=(\nu-2)^{2k+1}\,\frac{x}{2}\,I_{\nu-1}(x)+x\sum_{p_1=0}^{k}\binom{2k+1}{2p_1}S_{\nu-1,2p_1}(x).\end{aligned}$$

Sous la forme suivante on peut les regarder comme des formules récurrentes:

$$S_{\nu,2k+2}(x)=\nu^{2k+1}\,\frac{x}{2}\,I_{\nu-1}(x)-x\,S_{\nu+1,0}(x)-x\sum_{p_1=0}^{k-1}\binom{2k+1}{2p_1+2}S_{\nu+1,2p_1+2}(x),\quad.\;.\;.\quad(25)$$

$$S_{\nu,2k+2}(x)=(\nu-2)^{2k+1}\,\frac{x}{2}\,I_{\nu-1}(x)+x\,S_{\nu-1,0}(x)+x\sum_{p_1=0}^{k-1}\binom{2k+1}{2p_1+2}S_{\nu-1,2p_1+2}(x).\quad.\quad(25')$$

Chacune d'elles peut servir pour la construction d'une formule générale, qui représente la somme de la série $S_{\nu,2k+2}(x)$ à l'aide des séries

$S_{\nu+1,0}(x), S_{\nu+2,0}(x), \ldots S_{\nu+k+1,0}(x)$ resp. $S_{\nu-1,0}(x), S_{\nu-2,0}(x), \ldots S_{\nu-k-1,0}(x)$. Nous les supprimons, car pour μ arbitraire il n'est pas possible de représenter la somme de la série $S_{\mu,0}(x)$ par une forme finie.

En combinant les formules (25) et (25′), comme (9) et (9′) dans le paragraphe 3, nous trouvons les autres formules récurrentes:

$$\left.\begin{aligned} S_{\nu,2k+2}(x) = {} & \nu^{2k+1}\frac{x}{2}I_{\nu-1}(x) - x S_{\nu+1,0}(x) - \frac{x^2}{2}I_\nu(x)\sum_{p_1=0}^{k-1}\binom{2k+1}{2p_1+2}(\nu-1)^{2p_1+1} - \\ & - x^2 S_{\nu,0}(x)\sum_{p_1=0}^{k-1}\binom{2k+1}{2p_1+2} - x^2\sum_{p_1=0}^{k-2}\binom{2k+1}{2p_1+4}\sum_{p_2=0}^{p_1}\binom{2p_1+3}{2p_2+2}S_{\nu,2p_2+2}(x), \end{aligned}\right\} \quad (26)$$

$$\left.\begin{aligned} S_{\nu,2k+2}(x) = {} & (\nu-2)^{2k+1}\frac{x}{2}I_{\nu-1}(x) + x S_{\nu-1,0}(x) + \frac{x^2}{2}I_{\nu-2}(x)\sum_{p_1=0}^{k-1}\binom{2k+1}{2p_1+2}(\nu-1)^{2p_1+1} - \\ & - x^2 S_{\nu,0}(x)\sum_{p_1=0}^{k-1}\binom{2k+1}{2p_1+2} - x^2\sum_{p_1=0}^{k-2}\binom{2k+1}{2p_1+4}\sum_{p_2=0}^{p_1}\binom{2p_1+3}{2p_2+2}S_{\nu,2p_2+2}(x) \end{aligned}\right\} \quad (26')$$

De chacune de ces formules on peut déduire une formule générale qui représente la somme de la série $S_{\nu,2k+2}(x)$ seulement à l'aide des séries $S_{\nu+1,0}(x)$ et $S_{\nu,0}(x)$ resp. $S_{\nu-1,0}(x)$ et $S_{\nu,0}(x)$; aussi celles-ci nous supprimons pour le même motif.

Il est bien possible de représenter la somme de la série $S_{\mu,0}(x)$ pour μ entier par une forme finie en vertu des formules connues:

$$\sum_{n=0}^{\infty}(-1)^n I_{2n}(x) = \tfrac{1}{2}\{\cos x + I_0(x)\} \text{ et } \sum_{n=0}^{\infty}(-1)^n I_{2n+1}(x) = \tfrac{1}{2}\sin x,$$

des quelles on déduit entre autres:

$$S_{1,0}(x) = \sum_{n=0}^{\infty}(-1)^n I_{2n+1}(x) = \tfrac{1}{2}\sin x$$

et

$$S_{2,0}(x) = \sum_{n=0}^{\infty}(-1)^n I_{2n+2}(x) = I_0(x) - \sum_{n=0}^{\infty}(-1)^n I_{2n}(x) = \tfrac{1}{2}\{I_0(x) - \cos x\}.$$

Au moyen de celles-ci suivent de (26) pour $\nu = 1$ et de (26′) pour $\nu = 2$ après une légère réduction les formules récurrentes spéciales:

$$S_{1,2k+2}(x) = \frac{x}{2}\cos x - \frac{x^2}{2}\sin x\sum_{p_1=0}^{k-1}\binom{2k+1}{2p_1+2} - x^2\sum_{p_1=0}^{k-2}\binom{2k+1}{2p_1+4}\sum_{p_2=0}^{p_1}\binom{2p_1+3}{2p_2+2}S_{1,2p_2+2}(x), \quad (27)$$

$$S_{2,2k+2}(x) = \frac{x}{2}\sin x + \frac{x^2}{2}\cos x\sum_{p_1=0}^{k-1}\binom{2k+1}{2p_1+2} - x^2\sum_{p_1=0}^{k-2}\binom{2k+1}{2p_1+4}\sum_{p_2=0}^{p_1}\binom{2p_1+3}{2p_2+2}S_{2,2p_2+2}(x), \quad (27')$$

des quelles nous déduisons, en faisant différence entre k pair et k impair, les formules générales:

$$\left.\begin{aligned} & S_{1,4k+2}(x) = \sum_{n=0}^{\infty}(-1)^n(2n+1)^{4k+2}I_{2n+1}(x) = \\ & = \frac{x}{2}\Bigg[\cos x\left\{1 + \sum_{r=1}^{k}(-x^2)^r\sum_{p_1=0}^{2k-2r}\binom{4k+1}{2p_1+4r}\sum_{p_2=0}^{p_1}\binom{2p_1+4r-1}{2p_2+4r-2}\cdots\sum_{p_{2r}=0}^{p_{2r-1}}\binom{2p_{2r-1}+3}{2p_{2r}+2}\right\} - \\ & - x\sin x\sum_{r=0}^{k-1}(-x^2)^r\sum_{p_1=0}^{2k-2r-1}\binom{4k+1}{2p_1+4r+2}\sum_{p_1=0}^{p_1}\binom{2p_1+4r+1}{2p_2+4r}\cdots\sum_{p_{2r+1}=0}^{p_{2r}}\binom{2p_{2r}+3}{2p_{2r+1}+2}\Bigg] \end{aligned}\right\} \quad VIIa$$

$$\left.\begin{aligned}
&S_{1,4k+4}(x)=\sum_{n=0}^{\infty}(-1)^n(2n+1)^{4k+4}I_{2n+1}(x)=\\
&=\frac{x}{2}\Bigg[\cos x\Bigg\{1+\sum_{r=1}^{k}(-x^2)^r\sum_{p_1=0}^{2k-2r+1}\binom{4k+3}{2p_1+4r}\sum_{p_2=0}^{p_1}\binom{2p_1+4r-1}{2p_2+4r-2}\cdots\sum_{p_{2r}=0}^{p_{2r-1}}\binom{2p_{2r-1}+3}{2p_{2r}+2}\Bigg\}-\\
&-x\sin x\sum_{r=0}^{k}(-x^2)^r\sum_{p_1=0}^{2k-2r}\binom{4k+3}{2p_1+4r+2}\sum_{p_2=0}^{p_1}\binom{2p_1+4r+1}{2p_2+4r}\cdots\sum_{p_{2r+1}=0}^{p_{2r}}\binom{2p_{2r}+3}{2p_{2r+1}+2}\Bigg]
\end{aligned}\right\}\ \text{VII}$$

$$\left.\begin{aligned}
&S_{2,4k+2}(x)=\sum_{n=0}^{\infty}(-1)^n)\,2n+2)^{4k+2}I_{2n+2}(x)=\\
&=\frac{x}{2}\Bigg[\sin x\Bigg\{1+\sum_{r=1}^{k}(-x^2)^r\sum_{p_1=0}^{2k-2r}\binom{4k+1}{2p_1+4r}\sum_{p_2=0}^{p_1}\binom{2p_1+4r-1}{2p_2+4r-2}\cdots\sum_{p_{2r}=0}^{p_{2r-1}}\binom{2p_{2r-1}+3}{2p_{2r}+2}\Bigg\}+\\
&+x\cos x\sum_{r=0}^{k-1}(-x^2)^r\sum_{p_1=0}^{2k-2r-1}\binom{4k+1}{2p_1+4r+2}\sum_{p_2=0}^{p_1}\binom{2p_1+4r+1}{2p_2+4r}\cdots\sum_{p_{2r+1}=0}^{p_{2r}}\binom{2p_{2r}+3}{2p_{2r+1}+2}\Bigg]
\end{aligned}\right\}\ \text{VIII}a$$

$$\left.\begin{aligned}
&S_{2,4k+4}(x)=\sum_{n=0}^{\infty}(-1)^n(2n+2)^{4k+4}I_{2n+2}(x)=\\
&=\frac{x}{2}\Bigg[\sin x\Bigg\{1+\sum_{r=1}^{k}(-x^2)^r\sum_{p_1=0}^{2k-2r+1}\binom{4k+3}{2p_1+4r}\sum_{p_2=0}^{p_1}\binom{2p_1+4r-1}{2p_2+4r-2}\cdots\sum_{p_{2r}=0}^{p_{2r-1}}\binom{2p_{2r-1}+3}{2p_{2r}+2}\Bigg\}+\\
&+x\cos x\sum_{r=0}^{k}(-x^2)^r\sum_{p_1=0}^{2k-2r}\binom{4k+3}{2p_1+4r+2}\sum_{p_2=0}^{p_1}\binom{2p_1+4r+1}{2p_2+4r}\cdots\sum_{p_{2r+1}=0}^{p_{2r}}\binom{2p_{2r}+3}{2p_{2r+1}+2}\Bigg]
\end{aligned}\right\}\ \text{VIII}b$$

Pour $k=0$, $k=1$ et $k=2$ on obtient les formules particulières:

$$\sum_{n=0}^{\infty}(-1)^n(2n+1)^2I_{2n+1}(x)=\frac{x}{2}\cos x, \quad \ldots\ldots \tag{28}$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+2)^2I_{2n+2}(x)=\frac{x}{2}\sin x, \quad \ldots\ldots \tag{29}$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+1)^4I_{2n+1}(x)=\frac{x}{2}(\cos x-3x\sin x), \quad \ldots\ldots \tag{30}$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+2)^4I_{2n+2}(x)=\frac{x}{2}(\sin x+3x\cos x), \quad \ldots\ldots \tag{31}$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+1)^6I_{2n+1}(x)=\frac{x}{2}\{(1-15x^2)\cos x-15x\sin x\}, \quad \ldots \tag{32}$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+2)^6I_{2n+2}(x)=\frac{x}{2}\{(1-15x^2)\sin x+15x\cos x\}, \quad \ldots \tag{33}$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+1)^8I_{2n+1}(x)=\frac{x}{2}\{(1-210x^2)\cos x-x(63-105x^2)\sin x\}, \tag{34}$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+2)^8I_{2n+2}(x)=\frac{x}{2}\{(1-210x^2)\sin x+x(63-105x^2)\cos x\}, \tag{35}$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+1)^{10}I_{2n+1}(x)=$$

$$=\frac{x}{2}\{(1-2205x^2+945x^4)\cos x-x(255-3150x^2)\sin x\}, \quad (36)$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+2)^{10}I_{2n+2}(x)=$$

$$=\frac{x}{2}\{(1-2205x^2+945x^4)\sin x+x(255-3150x^2)\cos x\}, \quad (37)$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+1)^{12}I_{2n+1}(x)=$$

$$=\frac{x}{2}\{(1-21120x^2+51975x^4)\cos x-x(1023-41580x^2+10395x^4)\sin x\}, \quad (38)$$

$$\sum_{n=0}^{\infty}(-1)^n(2n+2)^{12}I_{2n+2}(x)=$$

$$=\frac{x}{2}\{(1-21120x^2+51975x^4)\sin x+x(1023-41580x^2+10395x^4)\cos x\}. \quad (39)$$

6. Ensuite nous examinons pour ν arbitraire et k entier $\geqq 0$ la série:

$$S'_{\nu,k}(x)=\sum_{n=0}^{\infty}(\nu+2n)^k I_{\nu+2n}(x). \quad \ldots\ldots \quad (40)$$

En faisant différence entre k pair et k impair nous trouvons, en appliquant la méthode des paragraphes précédents, les formules:

$$S'_{\nu,2k+1}(x)=\nu^{2k}\frac{x}{2}I_{\nu-1}(x)+xS'_{\nu+1,0}(x)+x\sum_{p_1=0}^{k-1}\binom{2k}{2p_1+2}S'_{\nu+1,2p_1+2}(x), \quad (41)$$

$$S'_{\nu.\,2k+2}(x)=-(\nu-2)^{2k+1}\frac{x}{2}I_{\nu-1}(x)+x\sum_{p_1=0}^{k-1}\binom{2k+1}{2p_1+1}S'_{\nu-1,2p_1+1}(x), \quad . \quad (41')$$

qui combinées conduisent aux formules récurrentes, que nous supprimons, parce que pour μ arbitraire la somme de la série $S_{\mu,0}(x)$ ne peut pas être représentée par une forme finie. Cela est bien possible dans le cas où μ est entier en vertu de la formule connue:

$$\sum_{n=0}^{\infty}I_{2n}(x)=\tfrac{1}{2}\{1+I_0(x)\},$$

de laquelle on déduit entre autres:

$$S'_{2,0}(x)=\sum_{n=0}^{\infty}I_{2n+2}(x)=\sum_{n=0}^{\infty}I_{2n}(x)-I_0(x)=\tfrac{1}{2}\{1-I_0(x)\}. \quad . \quad (42)$$

Posons dans (41) $\nu=1$ et dans (41′) $\nu=2$, nous trouvons selon (42) les formules spéciales:

$$S'_{1,2k+1}(x)=\frac{x}{2}\left\{1+2\sum_{p_1=0}^{k-1}\binom{2k}{2p_1+2}S'_{2,2p_1+2}(x),\right. \quad \ldots \quad (43)$$

$$S'_{2,2k+2}(x)=x\sum_{p_1=0}^{k}\binom{2k+1}{2p_1+1}S'_{1,2p_1+1}(x). \quad \ldots\ldots \quad (43')$$

En combinant ces deux formules à la méthode analogue à celle des paragraphes précédents, on obtient les formules récurrentes:

$$S'_{1,\,2k+1}(x)=\frac{x}{2}\left\{1+2\,x\sum_{p_1=0}^{k-1}\binom{2k}{2p_1+2}\sum_{p_2=0}^{p_1}\binom{2p_1+1}{2p_2+1}S'_{1,\,2p_2+1}(x)\right\},\,.\quad.\qquad(44)$$

$$S'_{2,\,2k+2}(x)=\frac{x^2}{2}\left\{\sum_{p_1=0}^{k}\binom{2k+1}{2p_1+1}+2\sum_{p_1=0}^{k-1}\binom{2k+1}{2p_1+3}\sum_{p_2=0}^{p_1}\binom{2p_1+2}{2p_2+2}S'_{2,\,2p_2+2}(x)\right\}.\qquad(44')$$

De ces formules on déduit ensuite les formules générales:

$$\left.\begin{aligned}S'_{1,\,2k+1}(x)&=\sum_{n=0}^{\infty}(2n+1)^{2k+1}\,I_{2n+1}(x)=\\&=\frac{x}{2}\left\{1+\sum_{r=1}^{k}x^{2r}\sum_{p_1=0}^{k-r}\binom{2\,k}{2p_1+2r}\sum_{p_2=0}^{p_1}\binom{2p_1+2r-1}{2p_2+2r-1}\cdots\sum_{p_{2r}=0}^{p_{2r-1}}\binom{2p_{2r-1}+1}{2p_{2r}+1}\right\}\end{aligned}\right\}\quad IX$$

$$\left.\begin{aligned}S'_{2,\,2k+2}(x)&=\sum_{n=0}^{\infty}(2n+2)^{2k+2}\,I_{2n+2}(x)=\\&=\frac{x^2}{2}\sum_{r=0}^{k}x^{2r}\sum_{p_1=0}^{k-r}\binom{2k+1}{2p_1+2r+1}\sum_{p_2=0}^{p_1}\binom{2p_1+2r}{2p_2+2r}\cdots\sum_{p_{2r+1}=0}^{p_{2r}}\binom{2p_{2r}+1}{2p_{2r+1}+1}\end{aligned}\right\}\quad X$$

Il est évident que les formules *IX* et *X* représentent la somme des séries

$$\sum_{n=0}^{\infty}(2\,n+1)^{2k+1}\,I_{2n+1}(x)\quad\text{et}\quad\sum_{n=0}^{\infty}(2\,n+2)^{2k+2}\,(x)$$

par une forme finie et bien par une polynôme de x du degré $2k+1$ resp. $2k+2$, dont les puissances de x sont impaires resp. paires.

On peut aussi, d'après un théorème de CAUCHY, développer en séries de puissances de x les membres gauches de *IX* et *X*, ainsi:

$$\left.\begin{aligned}S'_{1,\,2k+1}(x)&=\sum_{n=0}^{\infty}(2n+1)^{2k+1}\,I_{2n+1}(x)=\sum_{n=0}^{\infty}(2n+1)^{2k+1}\sum_{m=0}^{\infty}\frac{(-1)^m\left(\dfrac{x}{2}\right)^{2m+2n+1}}{m\,!\,(m+2n+1)\,!}=\\&=\sum_{r=0}^{\infty}\left(\frac{x}{2}\right)^{2r+1}\sum_{p=0}^{r}\frac{(-1)^p\,(2r-2p+1)^{2k+1}}{p\,!\,(2r-p+1)\,!},\end{aligned}\right\}(45)$$

et

$$\left.\begin{aligned}S'_{2,\,2k+2}(x)&=\sum_{n=0}^{\infty}(2n+2)^{2k+2}\,I_{2n+2}(x)=\sum_{n=0}^{\infty}(2n+2)^{2k+2}\sum_{m=0}^{\infty}\frac{(-1)^m\left(\dfrac{x}{2}\right)^{2m+2n+2}}{m\,!\,(m+2n+2)\,!}=\\&=\sum_{r=0}^{\infty}\left(\frac{x}{2}\right)^{2r+2}\sum_{p=0}^{r}\frac{(-1)^p\,(2r-2p+2)^{2k+2}}{p\,!\,(2r-p+2)\,!}.\end{aligned}\right\}(46)$$

L'égalisation des coëfficients de puissances homologues de x dans les deux membres de *IX* et *X* conduit aux identités importantes:

$$\sum_{p-0}^{r}\frac{(-1)^p\,(2r-2p+1)^{2k+1}}{p\,!\,(2r-p+1)\,!}=2^{2r}\sum_{p_1-0}^{k-r}\binom{2k}{2p_1+2r}\sum_{p_2=0}^{p_1}\binom{2p_1+2r-1}{2p_2+2r-1}\cdots\sum_{p_{2r}=0}^{p_{2r-1}}\binom{2p_{2r-1}+1}{2p_{2r}+1},\qquad(47)$$

donc en particulier $= 0$ pour $r > k$;

$$\sum_{p=0}^{r} \frac{(-1)^p\,(2r-2p+2)^{2k+2}}{p!\,(2r-p+2)!} = 2^{2r+1} \sum_{p_1=0}^{k-r} \binom{2k+1}{2p_1+2r+1} \sum_{p_2=0}^{p_1} \binom{2p_1+2r}{2p_2+2r} \cdots \sum_{p_{2r+1}=0}^{p_{2r}} \binom{2p_{2r}+1}{2p_{2r+1}+1}, \quad (48)$$

donc en particulier $= 0$ pour $r > k$.

De plus il est évident qu'on peut remplacer les formules IX et X par les autres:

$$S'_{1,\,2k+1}(x) = \sum_{n=0}^{\infty} (2n+1)^{2k+1} I_{2n+1}(x) = \sum_{r=0}^{k} \left(\frac{x}{2}\right)^{2r+1} \sum_{p=0}^{r} \frac{(-1)^p\,(2r-2p+1)^{2k+1}}{p!\,(2r-p+1)!}, \qquad IX'$$

$$S'_{2,\,2k+2}(x) = \sum_{n-0}^{\infty} (2n+2)^{2k+2} I_{2n+2}(x) = \sum_{r=0}^{k} \left(\frac{x}{2}\right)^{2r+2} \sum_{p=0}^{r} \frac{(-1)^p\,(2r-2p+2)^{2k+2}}{p!\,(2r-p+2)!}\ . \qquad X'$$

Les substituons $k = 0, 1, 2, \ldots 5$ dans IX et X ou dans IX' et X' donnent les formules particulières:

$$\sum_{n=0}^{\infty} (2n+1)\, I_{2n+1}(x) = \frac{x}{2}, \quad \ldots\ldots \quad (49)$$

$$\sum_{n-0}^{\infty} (2n+2)^2\, I_{2n+2}(x) = \frac{x^2}{2}, \quad \ldots\ldots \quad (50)$$

$$\sum_{n=0}^{\infty} (2n+1)^3\, I_{2n+1}(x) = \frac{x}{2}(1+x^2), \quad \ldots\ldots \quad (51)$$

$$\sum_{n=0}^{\infty} (2n+2)^4\, I_{2n+2}(x) = \frac{x^2}{2}(4+x^2), \quad \ldots\ldots \quad (52)$$

$$\sum_{n=0}^{\infty} (2n+1)^5\, I_{2n+1}(x) = \frac{x}{2}(1+10\,x^2+x^4), \quad \ldots\ldots \quad (53)$$

$$\sum_{n=0}^{\infty} (2n+2)^6\, I_{2n+2}(x) = \frac{x^2}{2}(16+20\,x^2+x^4), \quad \ldots\ldots \quad (54)$$

$$\sum_{n=0}^{\infty} (2n+1)^7\, I_{2n+1}(x) = \frac{x}{2}(1+91\,x^2+35\,x^4+x^6), \quad \ldots\ldots \quad (55)$$

$$\sum_{n=0}^{\infty} (2n+2)^8\, I_{2n+2}(x) = \frac{x^2}{2}(64+336\,x^2+56\,x^4+x^6), \quad \ldots\ldots \quad (56)$$

$$\sum_{n=0}^{\infty} (2n+1)^9\, I_{2n+1}(x) = \frac{x}{2}(1+820\,x^2+966\,x^4+84\,x^6+x^8), \quad \ldots \quad (57)$$

$$\sum_{n=0}^{\infty} (2n+2)^{10}\, I_{2n+2}(x) = \frac{x^2}{2}(256+5440\,x^2+23522\,x^4+120\,x^6+x^8), \quad (58)$$

$$\sum_{n=0}^{\infty} (2n+1)^{11}\, I_{2n+1}(x) = \frac{x}{2}(1+7381\,x^2+24970\,x^4+5082\,x^6+165\,x^8+x^{10}), \quad (59)$$

$$\sum_{n=0}^{\infty} (2n+2)^{12}\, I_{2n+2}(x) = \frac{x^2}{2}(1024+87296\,x^2+90112\,x^4+10032\,x^6+220\,x^8+x^{10}). \quad (60)$$

Biochemistry. — *On certain aromatic amines as histochemical reagents.* By H. G. DERX and G. H. OLTHETEN. (Laboratories of the "Unilever", Rotterdam.) (Communicated by Prof. L. G. M. BAAS BECKING.)

(Communicated at the meeting of November 28, 1942.)

C. WURSTER, in 1886, found a number of reagents on, what he called "active oxygen". Amongst those reagents were di- and tetra-methyl-p.-phenylenediamine, which now belong to the standard reagents in the qualitative research of oxidases. A great many other aromatic amines had already — and have since — been proposed for the localisation of these enzymes: sufficient to mention p.-phenylenediamine, also introduced by WURSTER.

They all have in common the property of being oxidisable and, thereby, to pass from a leucostage into a coloured condition. It is known that often this dehydrogenation may proceed in steps; one phase being reversible, the final stage, however, representing a stable pigment which cannot be easily reduced.

Several ways have been proposed to apply these reagents; Molisch, for instance, prepared, what he called "WURSTER's paper", which is filter paper soaked in a solution of tetramethyl-p.-phenylenediamine, which paper, when applied on a fresh plant section, shows the localisation of the oxidases by the formation of a blue colour. The reagents used by RACIBORSKI on his "leptomin" served a similar purpose. By means of these reagents often very striking pictures may be obtained.

Now WURSTER, in his paper on "Quantitative Bestimmung des Holzschliffes in Papier" noticed the curious fact, that the same reagents which he used on "active oxygen", yielded coloured products with wood-substance, i.e. with lignin. Long before 1887, however, aromatic amines had been used as reagents on lignification, as is illustrated in the following table, which may not even be exhaustive.

TABLE 1.
Aromatic amines as lignin-reagents.

No	Reagent	Author	Year	Colour
1	aniline	RUNGE	1834	yellow
2	pyrrole	RUNGE	1834	red
3	p-toluidine	RUNGE and HOFMANN	1865	yellow
4	xylidine	RUNGE and HOFMANN	1865	yellow
5	o-toluidine	RUNGE and HOFMANN	1865	yellow
6	indole	NIGGL	1881	red
7	β-naphtylamine	HARZ	1885	yellow
8	lepidine	IHL	1885	wine-red
9	skatole	MATTIROLO	1885	violet-red
10	carbazole	MATTIROLO	1885	violet-red
11	m-phenylenediamine	MOLISCH	1887	yolk-yellow
12	α-naphtylamine	WIESNER	1887	yellow
13	p-phenylenediamine	WURSTER	1887	orange (red?)
14	dimethyl-p-phenylenediamine	WURSTER	1887	fuchsia-red
15	tetramethyl-p-phenylenediamine	WURSTER	1887	dull-purple (red?)
16	diphenylamine	ELLRAM	1896	yellowish orange
17	toluenediamine	HEGLER	1899	orange
18	methylester of anthranilic acid	ERDMANN	1899	orange
19	phenylhydrazine	SENGT	1904	yellow

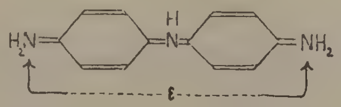
TABLE 1 *(continued)*.

No.	Reagent	Author	Year	Colour
20	p-phenylenenitrosoamine	BERGÉ	1906	lacquer-red
21	benzidine	SCHNEIDER	1914	yellowish orange
22	dimethylaminoazobenzene	PLAUT	1925	orange
23	quinoline	CASTIGLIONI	1935	wine-red
24	quinaldine	CASTIGLIONI	1939	yellowish green/green
25	1.4-aminoaphthol	DERX and OLTHETEN	1942	yellowish brown
26	4.4′-diaminodiphenylamine	DERX and OLTHETEN	1942	red-brownish red
27	2.4′-diaminodiphenyl	DERX and OLTHETEN	1942	yellowish brown

Before WURSTER, apparently, at least ten such amines had been used, many of them in acid solution.

As has been said before, these substances, all of them colourless, or almost so, in the reduced stage, form coloured compounds by oxidation. The lignin seems, therefore, to contain groups which are easily reduced. According to the well-known scheme of FREUDENBERG, the lignin chain contains various active groups, which might enter into reaction with amines. Our knowledge on this point is, however, rather scanty and it is not impossible that aromatic amines should be considered only as reagents on substances invariably accompanying lignin, like the reactions with phenols and polyphenols. Specific in a true sense of the word are only the reactions by which chlorinated lignin is formed (MÄULE, WOOD) and those reactions which are based on the adsorbtive power of lignin (BREGADZE, FRIESEN, GRÜSZ). Whatever this specificity, the cell wall, when it contains lignin, seems always to contain the accompanying substances as well.

Now in the course of an investigation on the distribution of oxidizing enzymes in plant-tissue, we had occasion to use a number of substances related to already well-known reagents. One relative of benzidine, 4.4′-diaminodiphenylamine, in an aqueous solution, containing 0.4 % of the hydrochloride, proved to be particularly suited, by its sensitivity as well as by the beautiful blue colour of its oxidation product, for the detection of oxidases as well as for the staining of lignified tissues. This blue oxidation product may be reduced again to the leucobase by means of such substances as ascorbic acid. Presumably the reversible blue stage designates the presence of a semiquinone of the substance, viz.



Beyond this blue oxidation-phase there is a stage reached in which the solution begins to yield a deposit of a brownish-purple tinge. This stage we shall designate as the irreversible oxidation-phase. This probably represents the formation of an indamine or its polymerization product.

$$NH_2—C_6H_4—NH—C_6H_4—NH_2 \quad —2H \rightarrow NH = C_6H_4 = N—C_6H_4—NH_2.$$

Very likely this oxidation product is formed when the compound reacts with lignin, skipping, however, the reversible blue stage. The blueish tinge, a peculiarity of solutions partly auto-oxidised is utterly lacking in the lignin reaction. The colour developed may be red, brownish red or brown. Whether the lignin itself influences the colour either directly or by changing the dispersion of a colloid, or whether other products present in the cell affect the colour remains to be investigated. Very young xylem cells in the stem of *Acer, Taxus, Mahonia, Chrysanthemum, Fragaria, Ranunculus, Ligustrum* and *Sambucus* show a red colouration with the reagent. With vanillin, first a yellow, later a reddish brown colour is developed. The shade is constant at p_H 1—8. Eugenol yields the same colour.

A further peculiarity is the speed of the reaction. This speed (as indicated by the colour developed) depends upon the p_H. Table 2 shows the reagent used simultaneously for the localisation of oxidase and lignin in cross-sections of the stem of *Taxus*, while table 3 shows the relation between p_H and speed of reaction.

TABLE 2.

Taxus baccata L., one-year-old twig. (cross section).

citric acid-phosphate buffers. Reagent: 4.4'-diamino-diphenylamine.

p_H	Xylem	Protoxylem	Medullary rays	Phloem.
(0,1 N HCl) 1,0	bright red	—	—	—
3,0	coral red	very faint red	—	after 30 min. greyish blue.
4,0	bright coral	very faint red	—	after 30 min. blue
5,0	brownish red	red	—	after 30 min. blue
6,0	brownish red	purple	light blue	dark blue
7,3	brownish red	blueish purple	light blue	dark blue
8,2	brownish red	blue	greenish blue	light blueish purple
(0,1 N NaOH) 13,0	yellow (effect of OH—)	—	—	—

TABLE 3.

Abies sp, slivers of heartwood.

citric acid-phosphate buffers. 4.4'-diamino-diphenylamine, 15° C.

p_H	Colour	Time required for optimal reaction
(0,1 N HCl) 1,0	red	30 minutes
2,2	red	7
3,0	red	6
4,0	red	5,25
5,0	brownish-red	2
5,6	brownish-red	2
6,0	brown, later yellowish-red	5
7,3	brownish-orange	15
8,2	yellow, later brownish	30
11,0	yellow colour (effect of OH—)	

When used in aqueous solution the reagent seems therefore, in a way, to be an indicator of the p_H of the cell wall as well. It appears that very young xylem-cells, in which the colour developed is bright red, must have a p_H of less than 5.0, while older xylem, in which the colour assumes a slightly brownish hue, is slightly less acid.

Repeating the experiment with *Abies*-wood at a higher temperature, it was shown that the reaction is influenced considerably by temperature, illustrating the fact that a true chemical reaction is going on. (See figure 1). At 100° C. the reaction develops its maximal colour within a few seconds. Treating the sections with the boiling reagent also has the advantage of inactivating the oxidases, thereby showing the distribution of lignin and the approximate p_H of its milieu, exclusively.

Hardwoods and softwoods yield the same colour with the reagent. The middle lamella of the cells of the phellem in sections of *Sambucus* stem showed a marked reaction. The compound is stable in the light, in glycerol jelly as well as in balsam. Alkali dissolves it out of the tissue, yielding a purple solution. It is stable in a weakly acid medium.

The reaction proceeds with the same velocity when oxygen is excluded, as was shown by experiments in vacuo.

The solution was easily sucked up by living stems of *Impatiens* and *Sambucus*. In 24 hours the xylem was coloured brownish-red, the surrounding phloem blue. The tissues died, however, by this treatment.

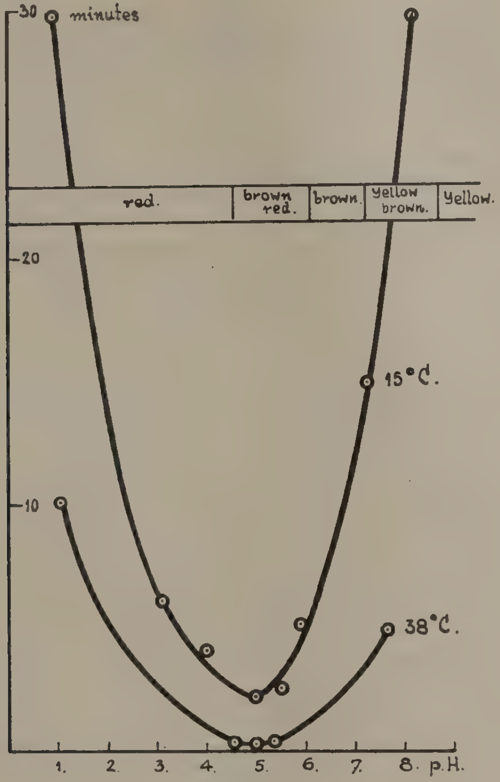


Fig. 1.

It appears that several other aromatic amines also show a colouring dependent upon the p_H. Table 4 shows the result.

To check the auto-oxidation of the reagent, the addition of 0.1 % of ammonium-thiocyanate to the solution is to be recommended. This substance will stop all heavy metal-induced oxidations, and the reagent will remain in good condition much longer.

Enzyme may also be inactivated by saturated aqueous sublimate. Here, however, a few local blue spots always develop, as sublimate is also an oxidizing agent.

TABLE 4.

Abies sp. slivers of heartwood.

Compound	p_H 1	p_H 5	p_H 12
2.4′-diaminodiphenyl	bright yellowish brown	yellowish brown	yellow
1.4-aminonaphtol	dirty yellow	yellow-orange	dirty yellow
p-phenylenediamine	yellow-orange	orange	yellowish green
tetramethyl-p-phenylenediamine	yellow	dirty purple	yellowish green

It is remarkable that WURSTER describes the colour with p-phenylenediamine as "red", and the tetramethyl-compound also as yielding a red colour with wood substance.

If, before the application of the reagent, a solution of Iodine and KI is used to inactivate the enzyme, an entirely different picture develops. Instead of a reddish brown colour the xylem is coloured bright- to yellowish-green. The phloem and the medullary rays show a blue colour (*Taxus*).

The vascular bundles in the stem of *Brassica oleracea, var. capitata* (cauliflower) when treated in this way, show the cells of the collenchymatous pericycle and of the phloem surrounded with light blue walls. The cells of the cambial region show greenish-grey walls, while the vessels show green in a peacock-green wood parenchyma.

Using o-tolidine instead of 4.4′-diaminodiphenylamine in conjunction with a preceding I—KI treatment more differentiation may be obtained.

Taxus baccata stems were used in transverse section. The phloem showed purplish-pink, the cambial zone pale pink, the newly formed xylem elements took a yellow tinge which, when the cells became older, veered over yellowish-green and green to blueish-green.

A pretreatment with bromine water, followed by diamino-reagent or tolidine gives a totally different picture. Here the xylem remains uncoloured, or takes a very feeble yellowish-green tinge. The phloem, however, shows brilliant green, like the medullary rays and the central pith.

The interpretation of these complicated phenomena is not easy.

It is probable that adsorption of I, resp. Br. plays a role in these reactions. However, BREGADZE claims that only sapwood adsorbs the iodine, whereas the reactions mentioned above were also obtained with heartwood. (*Fagus* and *Abies*).

We may assume that both the iodine and the bromine adsorbed oxidized the compounds, but that bromine, by its stronger action, actually destroys some of the compound by far-going oxidation. However, when the bromine water was diluted ten times, a similar result was obtained. The phloem remained colourless in this case.

A remarkable reaction was obtained in a transverse section of a cauliflower-stem. After a pretreatment in iodine the section was bathed in the diamino-reagent. The xylem coloured brown, which colour changed slowly to blue, first in the protoxylem, to spread, within half an hour, over the entire section.

Only the walls were coloured. The reaction did not show when the sections were placed previously in boiling water or when they were pretreated with a saturated aqueous solution of corrosive sublimate.

One may account for this phenomenon by the reputed presence of large quantities of ascorbic acid in the cauliflower-stem. This ascorbic acid will reduce the iodine, therefore prevent the green colour te develop, and, moreover, retard the further oxidation of the diamino-compound. As cauliflower possesses no oxidase, the action should be ascribed entirely to peroxidase.

From the substances studied, 4.4′-diamino-diphenylamine hydrochloride shows the most promise. For living as well as for permanent mounts it seems a most suitable reagent on lignification, state of the lignification and localization of oxidizing enzymes.

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Rotterdam, September-November 1942.

Botany. — *Aethyleen als stimulans bij het auxineverbruik bij erwten en tomaten.* Door JE. OORTWIJN BOTJES. (Communicated by Prof. W. H. ARISZ.)

(Communicated at the meeting of November 28, 1942.)

Inleiding.

Nadat NELJUBOV (1901) vond, dat de door WIESNER (1878) ontdekte horizontale krommingen bij verschillende kiemplanten veroorzaakt werden door sporen lichtgas en wel hoofdzakelijk door aethyleen, hebben verschillende andere onderzoekers nagegaan, waarop de invloed van het aethyleen berustte.

De uiterlijk zichtbare verschijnselen zijn bij kiemplanten van erwten volgens KNIGHT en CROCKER (1913) drievoudig. De „triple response" zou bestaan uit:

I. een remming van den lengtegroei;
II. een versterking van den diktegroei;
III. horizontale nutatie in de groeiende zône.

Een geremde lengtegroei, welke gepaard gaat met een toename van den diktegroei werd door VAN DER LAAN (1934) in aethyleen eveneens bij Avena waargenomen. Krommingen traden bij deze planten echter niet op.

Deze gedrongen groei van Avena in aethyleen werd door VAN DER LAAN toegeschreven aan de door dezen auteur geconstateerde verlaagde auxineproductie, waardoor een ophooping van voedingsstoffen in de coleoptielen zou ontstaan.

MICHENER (1938), die een ongewijzigde auxineproductie constateerde, schreef het effect van aethyleen enkel aan een ophooping of activeering van de voedingsstoffen toe.

ZIMMERMAN en WILCOXON (1935) toonden aan, dat verschillende verschijnselen, die in aethyleen bij tomaten en enkele andere planten optreden, zooals wortelvorming aan de bovenaardsche stengeldeelen en epinastie bij bladeren, eveneens opgewekt konden worden door verhooging van de groeistofconcentratie, b.v. door middel van heteroauxine.

CROCKER, HITCHCOCK en ZIMMERMAN (1935) concludeerden, dat aethyleen zich als een groeistof gedroeg en dus zelf een groeistof moest zijn.

BORGSTRÖM (1939) schrijft het effect van aethyleen toe aan een gewijzigd auxinetransport.

Daar het aethyleen schijnbaar invloed heeft op de totale auxineconcentratie of op de verdeeling van de auxine in de plant, werd door mij onderzocht of de door aethyleen bereikte effecten door middel van verhooging of van verlaging van de groeistofconcentratie konden worden opgewekt.

Alle proeven werden uitgevoerd in een donkere kamer. Eventueele horizontaal geplaatste erwten werden zoo geplaatst, dat ook de mediane vlakken horizontaal kwamen te staan.

Resultaten:

Proef I. *Erwten.*

Verlaging van de groeistofconcentratie.

Jonge erwtenkiemplanten waarvan het zaad verwijderd werd, waardoor de planten groeistofarm werden, werden in horizontalen stand geplaatst.

Op de geotropische prikkeling reageerden de epicotylen met een opwaartsche kromming.

Proef II. *Erwten.*

Verhooging van de groeistofconcentratie.

Afgesneden epicotylen van erwten werden op verschillend sterke heteroauxineoplossingen gebracht en horizontaal geplaatst. Op de geotropische prikkeling reageerden de planten met een verzwakte opwaartsche kromming (negatieve geotropie), met doorgroeien in het horizontale vlak (transversale geotropie) of met een kromming naar de aarde toe (positieve geotropie). Bovendien trad een zoodanige kromming op in het mediane vlak, dat de oorspronkelijk aan het zaad grenzende zijde convex werd. Deze kromming, die niet ontstaat tengevolge van de geotropische prikkeling, moet van autonomen aard zijn. Daar voor zoover wij weten tengevolge van de geotropische prikkeling de onderhelften steeds meer groeistoffen krijgen dan de bovenhelften zal de met heteroauxine verwekte positieve geotropie en de verzwakking van de negatieve geotropie moeten worden toegeschreven aan een groeiremming van de onderhelften der epicotylen door het overschrijden van de optimale groeistofconcentratie aldaar (BONNER 1933).

Proef III. *Erwten.*

Intacte planten in aethyleen.

Erwtenkiemplanten werden in horizontalen stand gebracht in verschillende concentraties aethyleen.

Bij de concentraties van 10^{-4} en 10^{-5} kromden de epicotylen naar de aarde toe (positieve geotropie), bij 10^{-6} groeiden ze door in het horizontale vlak (transversale geotropie) en bij nog zwakkere concentraties groeiden ze schuin naar boven. Het gedrag van planten in aethyleen is dus als bij proef II, dus alsof de groeistofconcentratie verhoogd en niet alsof ze verlaagd wordt.

Proef IV. *Erwten.*

Groeistofarme planten in aethyleen.

Zaadlooze en dus groeistofarme erwtenkiemplanten kregen bij een concentratie van 10^{-4} aethyleen in tegenstelling met de contrôles, die alle positieve geotropie kregen, negatieve geotropie, uitgezonderd 1 plant, die ongekromd gebleven was. Applicatie aan de topblaadjes van een groeistofhoudende pasta (1 % heteroauxine in lanoline), deed bij zaadlooze planten in aethyleen wel positieve geotropie ontstaan.

De geotropische reactie van epicotylen in een bepaalde concentratie aethyleen is dus, evenals in zuivere lucht afhankelijk van de sterkte van de aanwezige groeistofconcentratie.

Proef V. *Tomaten.*

Verlaging van de groeistofconcentratie.

Tomatenplanten worden tengevolge van een verblijf in het donker groeistofarm [ZIMMERMAN en HITCHCOCK (1936), OORTWIJN BOTJES (1938)]. Bij jonge tomatenplanten, die eenige dagen in het donker hadden gestaan, werd door mij soms hyponastie der bladeren waargenomen, doch nooit epinastie.

Proef VI. *Tomaten.*

Verhooging van de groeistofconcentratie en in aethyleen.

Jonge tomatenplanten vertoonden epinastie der bladeren zoowel bij verhooging van de groeistofconcentratie b.v. door middel van het opzuigen van een heteroauxineoplossing als in aethyleen.

Proef VI bevestigt de resultaten van ZIMMERMAN en WILCOXON. Uit de proeven V en VI blijkt tevens, dat ook tomatenbladeren zich in aethyleen gedragen alsof de groeistofconcentratie verhoogd en niet alsof ze verlaagd wordt.

Proef VII. *Tomaten.*

Groeistofarme planten in aethyleen.

Werden van eenzelfde serie tomaten, die in het donker stonden, op achtereenvolgende dagen enkele planten in aethyleen geplaatst, zoo bleek de optredende epinastie meer te verzwakken naarmate de planten langer in het donker hadden gestaan. Dit ging zoo voort tot er in het geheel geen epinastie meer optrad. De sterkte van de epinastie is dus evenredig met de in de bladeren nog aanwezige groeistofconcentratie.

Wanneer planten bij groei groeistof verbruiken, mag men aannemen, dat het groeieffect evenredig is aan het verbruik van de groeistof.

De groei zal dan afhangen van:

I. de hoeveelheid groeistof of de sterkte van de concentratie die in de plant aanwezig is;

II. de hoeveelheid groeistof, die de plant voor groei van een bepaalde sterkte verbruikt.

Deze hoeveelheid hangt af van specifieke eigenschappen van de plant, en wel van de volgende factoren:

A. het vermogen van de cel om groeistof te verbruiken; een actievere cel gebruikt van een zelfde groeistofconcentratie een grooter deel dan een minder actieve;

B. van de gevoeligheid voor groeistof, n.l. van het effect dat een bepaalde hoeveelheid verbruikte groeistof op de cel heeft. Een gevoeliger cel zal bij een zelfde groeistofverbruik grooter groeieffect hebben dan een minder gevoelige cel.

De groei van een cel hangt dus af:

1. van de sterkte van de groeistofconcentratie in het milieu, dus in de plant;
2. van de sterkte van het groeistofverbruik;
3. van de sterkte van den groei, die met een bepaalde hoeveelheid verbruikte groeistof correspondeert.

In aethyleen gedragen de epicotylen van erwten, zoowel als de bladeren van tomaten zich, alsof ze over meer groeistof beschikken. Dit kan een gevolg zijn van:

a. een verhoogde auxineconcentratie;
b. een verhoogd auxineverbruik;
c. een verhoogde gevoeligheid voor auxine.

Proef VIII. *Onderzoek naar een verhoogde auxineconcentratie.*

100 coleoptieltoppen van Avena ter lengte van 16 mm werden geplaatst in een concentratie van 10^{-4} aethyleen, de contrôles in lucht. Na 3 of 5 uren werden deze toppen in peroxyde-vrijen aether naar Utrecht [1]) gestuurd, waar de groeistofbepalingen plaats vonden. Het resultaat was, dat de aethyleen-topjes gemiddeld ongeveer 50 % minder auxine bevatten, dan de contrôles.

Hieruit blijkt, dat het effect van het aethyleen niet kan berusten op een verhoogde auxineconcentratie. Daar een verhoogde auxineconcentratie enkel tot stand kan komen door een verhooging van de auxineproductie of een remming van het auxinetransport, blijkt uit deze proef, dat er in aethyleen noch een verhooging van de auxineproductie, noch een remming van het auxinetransport optreedt.

Nu onder invloed van het aethyleen er geen verhooging van de auxineconcentratie plaats vindt, bevordert het aethyleen dus het groeistofverbruik of het maakt de planten gevoeliger voor groeistof. Uit de analyses bleek, dat er in korten tijd zeer veel groeistof vernietigd werd. Bij een zoo groote groeistofvernietiging is het onwaarschijnlijk, dat de gevoeligheid van de planten dermate verhoogd wordt, dat de planten zich gedragen, alsof er een verhooging van de groeistofconcentratie plaats heeft.

De afname van de hoeveelheid extraheerbare auxine, tengevolge van aethyleen is daarentegen een aanwijzing dat er meer auxine door de plant is verbruikt. Men mag dus aannemen, dat aethyleen het verbruik van auxine bevordert. Daar er tengevolge van een verhoogd verbruik minder auxine wordt afgevoerd naar verder van de productiecentra verwijderde weefsels zal de reactie van planten op aethyleen zijn een schijnbare remming van het groeistoftransport.

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[1]) Prof. Dr. V. J. KONINGSBERGER te Utrecht was zoo vriendelijk aan te bieden, de groeistofanalyses in zijn laboratorium te laten verrichten. Ook Dr. W. F. F. OPPENOORTH Jr. ben ik voor zijn medewerking zeer erkentelijk.

Dit onderzoek werd verricht in het Laboratorium voor Plantenphysiologie te Groningen.

Voor de hulp en de belangstelling bij dit onderzoek, ben ik Prof. Dr. W. H. ARISZ zeer dankbaar.

Summary.

The influence of weak concentrations of ethylene on the geotropic reactions of pea epicotyls and tomato leaves was studied. Cut off epicotyls placed in a strong heteroauxin solution and as well epicotyls of intact pea seedlings, placed in ethylene, undergo when laid down in a horizontal position a weakening of the negative geotropy or show transverse or positive geotropism.

Seedless pea seedlings, poor in growth substances, bend negatively geotropically in ethylene as well as in air.

Young tomato plants get epinasty of the leaves in ethylene or in sucking up a heteroauxin solution.

In the case of poverty of growth substance in consequence of having been in the dark for a long time, no epinasty will appear neither in air nor in ethylene.

Coleoptile tips of Avena 16 mm long contained about 50 % less auxin than the controls after having been for 3 or 5 hours in 10^{-4} ethylene.

Pea epicotyls and tomato leaves behave in ethylene as if the growth substance concentration is increased. This cannot be the consequence of a higher auxin production, no less than of a retarded auxin transport and may therefore be attributed to an increased auxin consumption.

Groningen, November 1942.

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Geology. — *On rocks from the isle of Batam (Riouw Archipelago).* By W. A. VAN DEN BOLD and J. P. VAN DER SLUIS. (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of November 28, 1942.)

In 1930 Dr. P. M. ROGGEVEEN carried out geological investigations in the Riouw Archipelago in charge of the Billiton Mij. His collections with his reports have been donated to the "Mineralogisch Geologisch Instituut" of the Utrecht University. The material of Batam and some surrounding islands has been examined by the authors.

The earliest publication about Batam is from BOTHÉ (1) who only mentions the granite E of Tering Bay with diabase and porphyries as younger products of the granitic magma. In a later, more extensive report (2) he also deals with other parts of the island. In NW Batam from Kp. Tg. Riouw (Senimba Bay) to Tg. Oetjang (2, p. 122) a strongly folded formation occurs consisting of "phyllitical clayshales". GISOLF (in 2) determined them as Liparites and Liparite-tuffs. The formation is entirely built up by volcanic material, a view which is wholly confirmed by our examination. The volcanics resemble the "Pahang Volcanic Series" from the Malayan Peninsula, ranging from Carboniferous to Triassic (2, p. 98), as has been also stated by ROGGEVEEN. This series has been found as far as S Batam where liparite-tuffs are outcropping near Tg. Goendak; it occurs also on the island of Boelan.

A second formation, called by ROGGEVEEN the "Central Batam formation" is found in Central Batam from the N to the S coast. It consists, according to BOTHÉ, of dark clayshales with plant-remains, pink and brown clayshales, pink micabearing claysandstones, talcous micasandstones, brown ironsandstones and white sandstones, dark blue-blackish sandstones, quartzites and quartz-conglomerates (2, p. 122—123). The dips in this formation are often steep, but generally less steep than in the Pahang Volcanic Series. Strata, comparable with the Central Batam formation are also found on the island of Singapore and, by means of fossils, have been determined there as Upper Trias. In conglomeratic sandstone of this formation at P. Samboe pebbles of radiolarian chert and granite have been found.

According to BOTHÉ (2) not all the sandstones of P. Batam are triassic. At different points on N Batam (Sg. Ladi and Tg. Pinggir), on S Batam (Sg. Piajoe) and on P. Awi arkosic sandstones, white sandstones and dark clayshales are found which are regarded to be of younger age, possibly cretaceous or tertiary and comparable with cretaceous sandstones or with the tertiary "plateau sandstone" of W Borneo. BOTHÉ mentions that LOTH and DE GROOT have stated that these younger sandstones rest with an angular disconformity on the triassic rocks (2, p. 99, 129—130); ROGGEVEEN, on the other hand, doubts the existence of this unconformity.

Some conglomerates, from P. Djandja Berias and P. Seraja are regarded by BOTHÉ (2, p. 123) to belong to the Quarternary. ROGGEVEEN mentions the occurrence of young conglomerates from P. Nongsa without giving his opinion on their precise age.

BOTHÉ distinguishes in NE Batam two different types of granite: the Batam- and the Bintan-type, the first with flesh-coloured Orthoclase and greenish Plagioclase, the second with colourless felspars. Though BOTHÉ and ROGGEVEEN did not find the contact of the granite with the triassic rocks they consider the granite to be younger which is in accordance with what is known from neighbouring regions. From P. Djoene ROGGEVEEN and BOTHÉ both mention a young effusive rock (Biotite Quartz Porphyry) which contains sharp-edged inclusions of granite and probably has genetic relations with this granite. According to ROGGEVEEN it is the quickly cooled top of the granite intrusion. ROGGE-

VEEN also found at this locality inclusions of a ?vitrified sandstone; this specimen is not present in our collection.

Pneumatolitic phenomena often occur in the granite: greisen have been found near Tg. Sabang and Tg. Bemban, quartz-tourmaline veins near Tg. Sabang, etc. Near Tg. Belah Molybdenite on jointplanes in granite has been found.

Near Tg. Boeton a greyish-black andesitic rock occurs. S of Tg. Boeton both BOTHÉ and ROGGEVEEN mention weathered porphyritical rocks, and the first considers them as a possible aequivalent of the Pahang Volcanic Series.

Description of rocksamples.

Granitic rocks. (758, 759, 760, 761, 762, 764, 769, 770, 773, 774, 775) [1].

We can distinguish two different types of Biotite granite.

1. The Batam granite with red Orthoclase, greenish Plagioclase and green Biotite (Pleochroic from yellow to dark green). The rock is medio- to coarse grained. Quartz is abundant in large bright crystals (undulous extinction hardly occurs), Orthoclase and Perthite predominate over Plagioclase (Albite): 758, 769, 775 (granite aplite in biotite granite). To the same type belong 770 (granite aplite) and 764 (aplitic granite porphyry). In 774 the Biotite becomes yellow-brown. This may be a transition to the Bintan type. The Biotite is often chloritized. Further occur: Epidote, Limonite, Magnetite, Apatite and Zircon. 770 (granite aplite) shows few Biotite, which is discoloured and chloritized. 775 (granite porphyry) possesses pink felspar-phenocrysts (Orthoclase) and some Albite and corroded Quartz in a groundmass composed of Quartz and acid felspar.

2. The Bintan type (759, 761, 762) is medio-fine grained and has colourless felspars. 759 and 761 show a porphyritical tendency, Molybdenite is deposited on jointplanes. Quartz and Orthoclase predominate, though there is somewhat more Albite than in the Batam granite. Perthite also occurs. The Biotite is pleochroic from yellow to dark brown (nearly opake), it is practically never chloritized. Few ore occurs. Further accessoria are Apatite and Zircon.

773 belongs to the Batam type, but forms a transition to greisen. It is a pneumatolitic alteration of the granite. The felspar has been partly changed into colourless mica, but Orthoclase and Albite are still present between the large Quartzes. Dark minerals have been altered into Chlorite and Epidote, Magnetite into Limonite. Fluorite occurs throughout the slide. Tourmaline is crystallized in a cavity.

Greisen (755, 756, 757, 772). They are gray rocks with large Quartz-crystals in a sort of groundmass of lightgreen mica, which appears to be colourless in the slide. The mica forms relic-structures of felspars which often have been idiomorphic. The Quartz forms large crystals or small intergrown ones. In 755 there occurs a remnant of Biotite. In 755 and 772 much Fluorite occurs; in the last one in veins.

771 from Tg. Sabang is part of a Tourmaline-Quartz vein. The Tourmaline-needles are pleochroic from pale brown to nearly black; partly blueish.

On the East coast of P. Nongsa a peculiar rock has been found (767, 768). It proved to be a granite which shows alteration to an arkose-like rock. In a dirty-white powdery matrix lie Quartzgrains. The matrix consists of Orthoclase and Albite. In the slide large crystals of Quartz, Albite and Perthite can be seen. The Quartz is hardly undulous. Some felspars are sericitized.

Near Tg. Toeroet a propylitic rock occurs which possibly also has been a granite (316, 317). It is a greenish rock with much Limonite. Quartz is present in large crystals and in small ones; sometimes the large crystals have the same optical orientation over some distance. The felspars have been entirely sericitized and silicified. Biotite is partly chloritized; at the margin it possesses a black-brown colour, in the centre a green one (pleochr.

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[1]) The numbers of the samples refer to those of the year catalogue of 1941 of the "Mineralogisch-Geologisch Instituut" at Utrecht and to those on the map.

yellow-brown to dark green). The ore is Pyrite, often limonitized. Limonite also occurs along cracks. Further some Leukoxene and some Zircon are present.

Porphyritical rocks from P. Djoene. The Quartzporphyri(t)es from P. Djoene (277, 281, 282, 283, 284, 286, 287) are dark gray to black, grayish-brown-wheathering rocks with small (2 mm large) felspar-phenocrysts and Quartz. Many large black inclusions occur. They are hypocrystalline rocks with sericitized felspar-phenocrysts with some Epidote, Calcite or Prehnite and Quartz-phenocrysts which are strongly corroded. The only dark mineral is entirely chloritized Biotite with some fibrous Epidote. In some cases the felspar could be determined; in 287: Oligoclase and in 283: Albite. The groundmass is devitrified and contains splinters of Quartz. It shows flowing-structure. Locally silicification is present; further there occur Magnetite with Limonite; sometimes Apatite and Zircon. A part of 281 consists of large Quartzes and fine Quartzgrains (silicification) with much Epidote, Chlorite and with Fluorite. In these porphyrites many inclusions are found:

1. Granite (in 277, 282). Intergrowth of large, bright Quartzes — hardly undulous — and Orthoclase is to be seen. Sometimes granophyr is present; slightly sericitized Albite equally occurs. There may be relics of Biotite in Chlorite and Epidote.
2. Sandstone (277). A small inclusion wholly consisting of Quartz with quartzitic texture.
3. Porphyrites. These inclusions are abundant. They may or may not contain phenocrysts (Oligoclase in 277). The groundmass consists of non-orientated Oligoclase and much Magnetite. The felspars show single twinning but no lamellation. Further Chlorite and Epidote occur.
4. Amphibole Porphyrites (287). The felspar phenocrysts are strongly sericitized; some Amphiboles (green, hardly pleochroic) occur; this mineral is also found in streaks in the groundmass. Much Epidote. The, probably silicified groundmass consists of small Quartzgrains.
5. In 283 we find an inclusion, consisting of Chlorite and Epidote with large quantities of Calcite and Fluorite. The origin is not known, but it may be derived from an unknown calcareous sediment.

286 is a quartzporphyrite which is strongly limonitized. The phenocrysts of felspar are probably kaolinized; there occur corroded Quartzes.

285, from P. Djoene, is a strongly altered quartzporphyrite. The original nature of the rock is still recognizable by the presence of Quartz-phenocrysts agreeing fairly well with those in the other quartzporphyrites. The groundmass has, however, been changed into an aggregate of small Quartzes and large crystals of Epidote and Prehnite; in the slide there is also a veinlet of Epidote.

As we have seen ROGGEVEEN is of the opinion that the quartzporphyrites of P. Djoene are a roof-intrusion of the granitic magma. From the foregoing description it is clear that the microscopical habit does not at all support this contention. The only admissible conclusion is that the porphyrites which contain inclusions of granite are younger than the granite and that for the rest nothing can be said of the relation of their magma with the granitic magma.

Pyroxene Porphyrite; 323 and 766 (pebble).

Dark gray, locally greenish rocks, in which we can recognize small phenocrysts of Pyroxene. In a groundmass of finegrained felspar (Andesine), silicified in 766, and small crystals of Magnetite lie large phenocrysts of basic plagioclase (often zonal with a core of Labrador) and colourless Pyroxene (Magnesia-Diopside). Both phenocrysts may be chloritized and in 766 silicified. Probably also Biotite has been present, but this has been changed completely into Chlorite and Epidote. Accessoria are Apatite, Sphene and Zircon. In 323 a rhombic Pyroxene also occurs in the groundmass. Here we also find an endogeneous inclusion. Magnetite also occurs in larger crystals.

Quartzporphyri(t)es 54, 55, 63, 66, 69, 70, 71, 72 (pebble from conglomerate), 291

(P. Koebong), 296, 305 (pebble), 306 (pebble), 311 (East Coast), 314 (East Coast), 315 (East Coast), 318, 636 (pebble from conglomerate), 638 (idem).

Gray-white and violet to purplish red rocks, strongly altered; some becoming schistose or phyllitic. The texture is porphyritical. The felspar-phenocrysts have been altered so strongly, that their original composition could not be determined. They are sericitized (63, 318, 636, 638), kaolinized (54, 66, 314, 315) or silicified (55, 69, 70, 71, 72, 296, 305, 306). The groundmass is finegrained, silicified or seriticized. 70, 71, 291 and 311 show hydrargillitization; 291 and 311 from the E coast show an entirely hydrargillitized matrix in which lie large and capriceously-shaped Quartzes. The Hydrargillite seems to replace parts of the Quartz (Compare: HERMES and DE VLETTER: lit. 5, fig. 1). 291 is a chalk-white rock. All rocks (exc. 291) contain abundantly Limonite, Magnetite and Leucoxene. 55 and 63 are schistose.

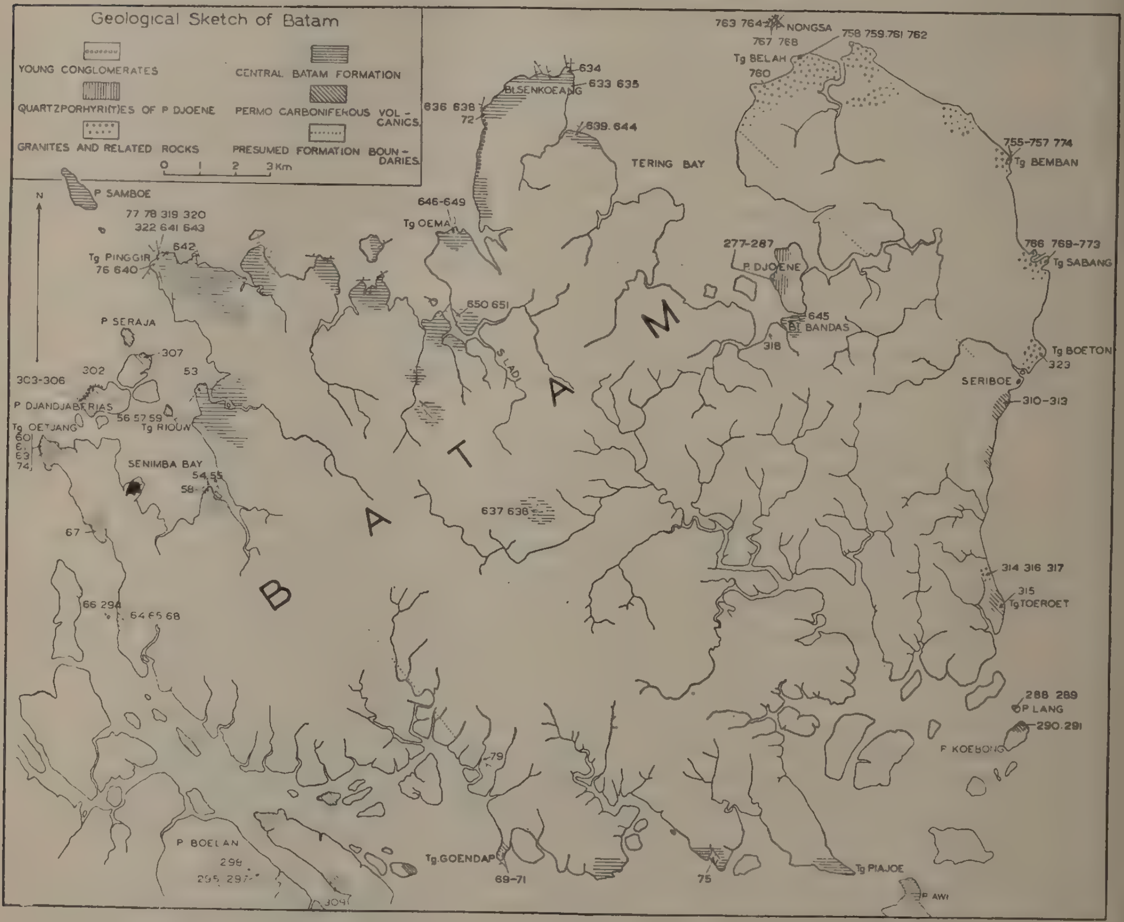

Fig. 1.

The rocks 54, 55, 63, 66, 69, 70, 71, 296 belong to the sequence of W Batam, regarded as the aequivalent of the Pahang Volcanic Series; 72, 305, 306, 636 and 638 are pebbles from conglomerates; 311, 314 and 315 are from the E coast of Batam, where, according to BOTHÉ the Pahang Volcanic Series is equally outcropping; 291 is from P. Koebong, E Batam; 318 has been found on the N coast of Batam, where ROGGEVEEN only indicates the Central Batam Formation.

Quartzporphyritical tuffs. 59, 64, 65, 67, 74, 297, 298, 633, 635, 646—649 (pebbles).

Gray and violet, strongly schistose rocks. Two tuffs are agglomeratic (67, 635). The felspars are wholly sericitized; Quartz occurs as corroded phenocrysts and as splinters. The groundmass is devitrified (633), silicified and sericitized (65, 74, 297, 638, 646—649); 298 is hydrargillitized. The Sericite of the groundmass often shows foliation (59, 64, 65, 74). 64 is a phyllitical rock containing Quartz, Sericite and Talc. 648 is a tuff in contact with an arkose.

The rocks 59, 64, 65, 67, 74, 297 and 298 belong to the Pahang Volcanic Series of W Batam; 633 and 635 are from Bt. Senkoeang in N Batam, where ROGGEVEEN only indicates the Central Batam Formation.

Porphyrites. 56, 302, 303, 309.

Dark brown-gray, white or violet rocks. Schistose. The felspar-phenocrysts are sericitized, the groundmass is silicified and sericitized. 302, 303, 309 resemble a sericite schist. All the rocks are from the Pahang Volcanic Series in W Batam.

Quartzporphyrites 68, 294.

Gray (294) or violet, altered (68) rocks with Quartz- and felspar-phenocrysts. The Quartz is idiomorphic and corroded. The felspars range in composition from Albite-Oligoclase to Oligoclase and show the beginning of sericitization. In 294 they are strongly fractured and occur in large fragments and small splinters. The groundmass is silicified; in 68 twinned Plagioclase laths can be distinguished. Apatite, Zircon, Magnetite, Leucoxene and Limonite occur. In 294 Chlorite occurs pseudomorphic after Biotite. Magnetite mostly occurs in the felspars and is often associated with some Leucoxene. The rocks 68 and 294 come from the Pahang Volcanic Series of W Batam.

Arkoses, quartzitic-arkoses, sandstones and quartzites form transitions; the boundary between them mostly is not very sharp. The occurrence of Zircon in all rocks, and of Tourmaline in nearly all (exc. 637, 639, 643, 645, 650), points to the origin from the same motherrock.

Arkoses. (53, 76, 77, 79, 642, 643, 645, 651).

Gray sandy rocks, sometimes coloured red-brown by Limonite (77, 79, 642); sometimes the Limonite along shearplanes (77). Sandy arkoses (77, 79, 651) as well as quartzitic ones occur (53, 76, 642, 645). In 79, 651 the Quartz is undulous; 79 moreover contains bent Muscovite. The felspars are sericitized (53, 77, 642, 645, 651) or sericitized and kaolinized (76, 79). All arkoses contain rounded grains of Zircon and Tourmaline (exc. 645). Limonite occurs in all samples, Magnetite in nearly all. 651 is different in so far as it contains remnants of plants and decoloured Biotite.

Quartzitic sandstones and quartzites (634, 637, 639, 640, 650).

Gray-white rocks, sometimes brown-coloured by Limonite. In 634 the Quartz is fractured and all the rocks show more or less undulous extinction of the Quartz (strong in 634, 637, 650). Moreover 637 shows strings (often parallel) of small inclusions in the Quartz which pass uninterrupted from one grain to another. 637 and 640 contain Zircon as well as Tourmaline, 637, 639, 650 only Zircon. 639, 640 contain Muscovite. They all contain Limonite and nearly all Magnetite and Leucoxene.

Argillaceous sediments. 60, 61, 75, 78, 304 (pebble), 319, 320.

Dark-gray argillaceous (319, 320), red-brown platy (60) pink-yellow to violet schistose (61, 75) and silicified (304) rocks. Microscopically they consist of argillaceous minerals with Quartz and Limonite (all samples), Sericite (75, 304) and Kaoline (61). Magnetite is abundant in 75. We may call 60 a lateritized clay-slate, while 61, 75, 319, 320 are lateritized clay-shales. 304 moreover is silicified and sericitized. 78 is quite different: a gray-blueish, finegrained rock with crossbedded yellow layers; on the beddingplanes we see Muscovite. It is an argillaceous-micasandstone. Microscopically the rock consists of Quartz and Muscovite. The Quartz is mostly not rounded. In fissures Hydrargillite occurs.

All the arkoses, quartzitic arkoses, quartzitic sandstones and quartzites and most of the argillaceous rocks have been found in the Central Batam Formation; only 60 and 61 are from the Pahang Volcanic Series; they differ from the other argillaceous rocks by their strong fissility.

Kaoline rocks. 312, 313 (East Coast of Batam).

They consist of Kaoline. The rocks show some bedding. Further only occur some Quartz, Leucoxene and Sericite; the samples have not been taken from the solid rock; they are isolated fragments (? pebbles).

Sericite and Quartz-sericite rocks. 290 and 57.

290 is made up of fine Sericite-flakes, Limonite and some Haematite, and Magnetite. Few, very fine, needles of Rutile occur. 57 consists of sharp-edged grains of sericitized material and Quartz embedded in a matrix of Limonite. The rocks give the impression of belonging to the Pahang Volcanic Series.

Limonite (288, 289, 322).

The rock consists of Limonite. In those of Lepang some Haematite occurs. In 322 and 288 some Quartz is present.

Quartzbreccias (307), *quartzconglomerate* (293, 641) and *Reefquartz* {58, 295, 644 and 763 (pebble)}.

The quartzbreccias are cataclastic and cemented by Limonite and silica. In 641 the Quartz is somewhat rounded and fractured. The reefquartz is monomineral.

A strongly *hydrargillitized and tourmalinized quartzose rock* (310) has been found as a pebble, together with the above-mentioned pebbles of kaoline rock. It resembles by its hydrargillitization strongly the rocks 291 and 311, but it does not show a porphyritical structure. Its original status cannot be stated.

Summary.

1. It is a pity that no contacts between the granites and the Central Batam Formation have been sampled. In consequence, we only have few arguments for stating their relative age. The presence of allothigenous Tourmaline in the arkoses of the Central Batam Formation may give the impression that the granite is older than the sediments. The general opinion about their relative age is just the reverse; this may be supported by the fact that in the granite the Quartzes do not show undulous extinction or cataclasis whilst the Quartz of the arkoses etc. always shows undulous extinction.

2. The Pahang Volcanic Series of W Batam which seems to occur also on East Batam (Samples 290, 291, 314, 315) is the oldest formation of the island. It has been more strongly folded and metamorphosed than the Central Batam Formation.

3. The Central Batam Formation consists of arkoses, quartzites and argillaceous sediments which have been possibly derived from an older granite (occurrence of Tourmaline and Zircon in almost every sample!) At Tg. Oema a pebble of quartzporphyrite has been found in an arkose; this proves once again that the Central Batam Formation is younger than the Pahang Volcanic Series. It is noticeable that from W of Batoe Bandas (318) we determined a quartzporphyrite, where ROGGEVEEN only mentions sandstones. According to his sections it is possible that an outcrop of Permo-Carboniferous occurs here. This would be also in accordance with his opinion that at Bt. Sengkoeang (roughly in the strike) the base of the sandstones crops out.

4. In NE Batam the granite crops out. We could recognize the two different facies (Batam and Bintan type) distinguished by older authors. As some transitions occur it is clear that they belong to the same magma. The strongly crumbled rock from P. Nongsa gives the impression of a granite passing "in situ" into an arkose. As no undulous extinction of the Quartz occurs and as the praetriassic granite described by GISOLF (in BOTHÉ, 2) from P. Samboe shows strong shearing, the arkose-granite from P. Nongsa must be younger than that of P. Samboe. It is probably of the same age as that of Batam and Bintan and thus represents a much altered facies of it. The existence of a granite in two completely different facies is in accordance with the views held by WESTER-

VELD (4) and an argument against WING EASTON who assumed that a younger granite intruded into a much altered older one in the larger Tin islands.

5. The quartzporphyri(t)es of Djoene represent according to ROGGEVEEN the quickly cooled top of the granite or a roof-intrusion. We have seen that the samples do not present any argument in favour of this opinion.

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4. J. WESTERVELD, Economic Geology, **32** (1937).
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Palaeontology. — *Youngtertiary smaller foraminifera from the neighbourhood of Ngimbang, East Java.* By J. P. VAN DER SLUIS and D. R. DE VLETTER. (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of November 28, 1942.)

The material dealt with in this publication was collected by Prof. L. RUTTEN in the neighbourhood of Ngimbang, residency of Surabaya, East Java. The four localities, from where samples have been studied, are:

1. Ngimbang,	6 km. N	from the S.W. corner of sheet no. 109 (Lamongan) of the geological map of Java, 1 : 100.000, 1938.
	8 km. E	from idem.
2. Gg. Girik,	8 km. N	,, ,,
	7 km. E	,, ,,
3. Wotan,	5,5 km. N	,, ,,
	3,5 km. E	,, ,,
4. Primpen,	4,5 km. N	,, ,,
	1 km. E	,, ,,

The Foraminifera suggest a young tertiary age of the formation, which on the geological map of Java, 1 : 100.000 of 1938, also has been indicated as Pliocene. 80 of the 129 species are also found in recent seas, 4 new species and 2 new varieties are described. The species are compiled in the subjoined list, on which is also indicated where they have been found elsewhere in the Netherlands East Indies.

The aberrant and new forms are described below.

Clavulinoides javanicus n. sp. fig. 1*a, b*. 46 ×.

Test about 2½ times as long as broad, elongate, sides parallel in the adult, chambers distinct, sutures very broad and slightly if at all depressed, wall finely arenaceous. Aperture a rounded opening at the end of a slight projection; keeled.

The megalospheric form is 1½ times as long as broad.

Resemblance with *C. aspera* (CUSHM.); distinctive are the very broad sutures.

Marginulina costata (BATSCH) var.

This specimen differs from the typical in having no costae on the last chamber, just as *Marginulinopsis densicostata* THALMANN.

Nodosaria badenensis (D'ORB.) var.

The holotype of REUSS (Versteinerungen Boehm. Kreide) is totally smooth and polished, where our specimen is roughly finished and of about half the size.

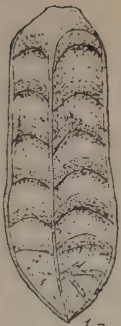
? Frondicularia spinosa n. sp. fig. 2. 36 ×.

The broad test much compressed with larger and minor spinose processes, initial end inflated. The test is irregularly tapering towards the, somewhat curved, initial end. Chambers distinct, the earlier ones slightly coiled, then uniserial. Sutures distinct and limbate. Wall smooth, hyaline and ornamented with numerous raised striae. A part of the apertural end of the holotype is broken off.

Length: 0,8 mm.

Nonion multicameratum n. sp. fig. 3*a, b*. 44 ×.

Test involute, bilaterally symmetrical and compressed; the umbilici depressed. Periphery subacute, chambers distinct and numerous, 16 to 17 in the last whorl, they are of a rather uniform shape and size, sutures slightly curved and limbate; wall finely perforated; aperture formed by an indistinct narrow slit at the base of the apertural face.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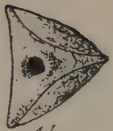
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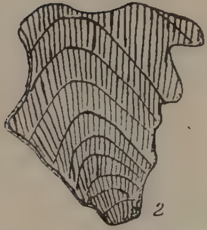
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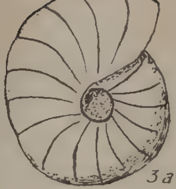
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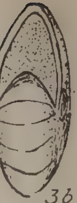
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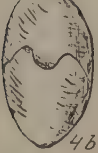
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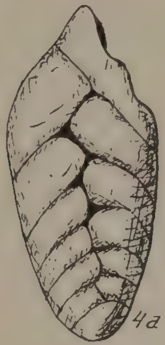
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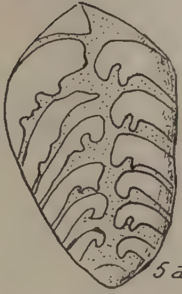
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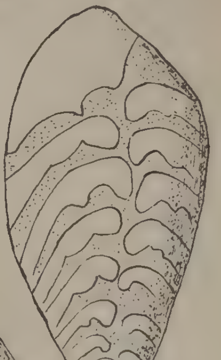
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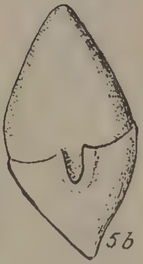
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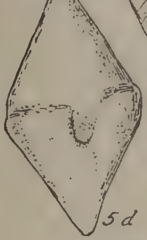
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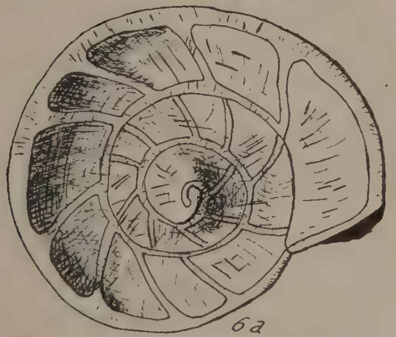
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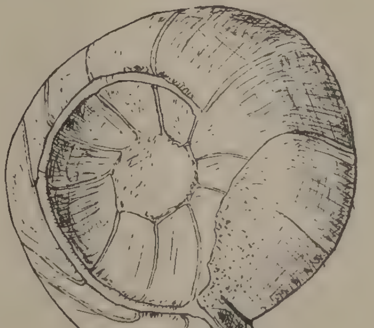
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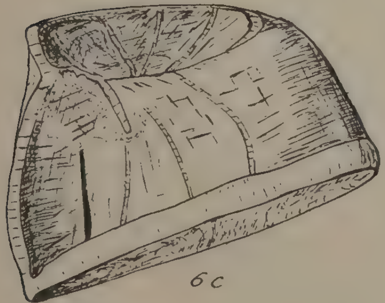
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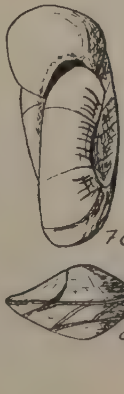
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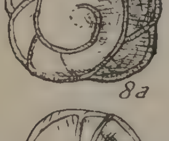
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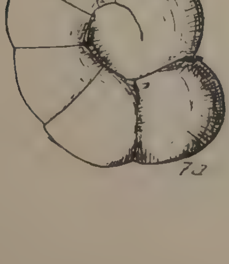
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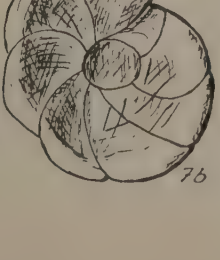
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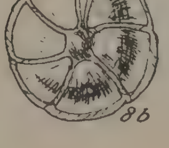
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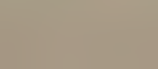

8c

SMALLER FORAMINIFERA FROM THE NEIGHBOURHOOD OF NGIMBANG	Localities	Kabu, Surabaja. (Koch)	East Borneo. (Koch)	Sonde and Modjokerto. (Boomgaart & Vroman)	Central Sumatra. (Le Roy)	East Ceram. (Koch & Fischer)	Bantam. (Yabe & Asano)	Upper Cretaceous	Eocene	Oligocene	Miocene	Pliocene	Recent
Clavulinoides javanicus v.d.Sluis & de Vletter n.sp.	1,2,4.												
- szaboi (Hantken)	1,2.		x						—	—			
Karreriella baccata (Schw.)	3.		x						—	—	—	—	—
- syphonella (Reuss)	1,2.		x					—	—	—	—	—	—
cf. Massilina decorata Cushm.	3.								—	—			
Spiroloculina depressa d'Orb.	3.			x			x			—	—	—	—
Vertebralina sp.	2.												
Cristellaria (R) calcar (Linn.)	1.	x	x		x	x	x		—	—	—	—	—
- (R) costata (Fichtel & Moll)	1.			x							—	—	—
- (R) - - var. subdecorata Cushm.	3.												—
- (R) crassa d'Orb.	1,2.		x					- -	—	—	—	—	—
- cultrata (Montfort)	2,4.		x			x		—	—	—	—	—	—
- (R) echinatus (d'Orb.)	3,4.		x								—	—	—
- (R) gyroscalprum Stache	2.								—	—	—		
- (R) iota Cushm.	1,2.			x				—	—	—	—	—	—
- (R) cf. lucida Cushm.	1.			x						—	—	—	—
- (R) orbicularis (d'Orb.)	3.		x			x		—	—	—	—	—	—
- rana Koch	1.		x							? - -	—	- - ?	
- rotulata (Lam.)?	3.		x	x		x		—	—	—	—	—	—
- (R) submamilligera Cushm.	4.								—	—	—	—	—
- (R) vortex (F. & M.)	4.			x				—	—	—	—	—	—
Planularia gemmata (Brady)	1,2.			x?		x						- -	—
Marginulina costata (Batsch) var.	3.												
- subaculeata (Cushm) var. glabrata (Cushm.)	1,2.								—	—	—	—	—
Dentalina consobrina d'Orb.	3.		x		x?	x		—	—	—	—	—	—
- - var. emaciata (Reuss)	3.	x	x	x			x		—	—	—	—	—
- coopereensis Cushm.	2.								—	—			
- floscula d'Orb.	1.										—		
- quadrulata Cushm. & Laiming	3.										—		
Nodosaria ambigua (Neugeboren)	3.								—	—	—	—	- -
- badenensis d'Orb. var.	3.												
- hirsuta d'Orb.	1.							—	—	—	—	—	—
- insecta (Schw.)	2,3.	x	x						—	—	—	—	—
- oligostegia Reuss	3.							—	—	—	—	—	—
- vertebralis (Batsch)	2.	x?	x	x		x		- -	—	—	—	—	—
Saracenaria acutauricularis (F.& M.)	1,2.							—	—	—	—	—	—
Vaginulina ?legumen Linné	1.							—	—	—	—	—	—
- ?spinigera Brady	2.								—	—	—	—	—
Frondicularia advena Cushm.	3.								—	—	—	—	—
?Frondicularia spinosa v.d.Sluis & de Vletter n.sp.	1,2.												
Lagena hispida Reuss	3.	x				x		—	—	—	—	—	—
- laevis (Mont.) var. nebulosa (Cushm.)	2.												—
- marginata (Walker & Boys)	3.	x	x				x	—	—	—	—	—	—
- - var. carinata Wiesner	3.												—
- orbignyana Seguenza	3.		x				x	—	—	—	—	—	—
- striata d'Orb.	3.		x				x		—	—	—	—	—
Nonion multicameratum v.d.Sluis & de Vletter n.sp.	3.												
Spiroplectoides cubensis Cushm. & Berm.	4.								—				
Bolivinita quadrilatera (Schw.)	1,2.				x							—	—
Plectofrondicularia californica Cushm. & Stewart	2,3.									—	—	—	—
- interrupta (Karrer)	2.		x						—	—	—	—	—
- miocenica Cushm.	3.									—	—		
- vaughani Cushm.	1,2,3.							—	—	—	—		
Bulimina affinis d'Orb.	1.		x					—	—	—	—	—	—
- ovata d'Orb.	2,3.		x					—	—	—	—	—	—
- pupula Stache	2.								—				
- striata d'Orb.	1,3.								—	—	—	—	—
Virgulina acuta (d'Orb.)	1,2.										—		
- schreibersiana Czjzek	1,2.							—	—	—	—	—	—
Bolivina alata (Seguenza)	2.				x					—	—	—	—
- antigua d'Orb.	2.								—	—	—	—	—
- arta Macfadyen	1.									—	—		
- cf.budensis Hantken	3.								—	—	—	—	—
- catanensis Seg.	2.											—	—
- dilatata var. javana v.d.Sluis & de Vletter n.sp.	1.												
- scalprata Schw. var. miocenica Macfadyen	2,3,4.										—		
- uniforminata Le Roy	1.				x						—		

SMALLER FORAMINIFERA FROM THE NEIGHBOURHOOD OF NGIMBANG	Localities	Kabu, Surabaja. (Koch)	East Borneo. (Koch)	Sonde and Modjokerto. (Boomgaart & Vroman)	Centraal Sumatra. (Le Roy)	East Ceram. (Koch & Fischer)	Bantam. (Yabe & Asano)	Upper Cretaceous	Eocene	Oligocene	Miocene	Pliocene	Recent
Rectobolivina bifrons Brady	3.											—	—
- - var. striatula (Cushm.)	1.											—	—
Loxostoma karrerianum (Brady)	4.											—	—
Uvigerina crassicostata Schw.	1,2.	x							—	—	—	—	
- farinosa Hantken	1.			x					—	—	—		
- mediterranea Hofker	1,4.			x					—	—	—	—	—
- multicostata Le Roy	2,4.				x						—		
- pigmaea d'Orb.	4.	x	x						—	—	—	—	—
- proboscidea Schw.	1,2,4.										—	—	
Siphogenerina dimorpha (Parker & Jones)	3.	x							—	—	—	—	—
Siphonodosaria fijiensis Cushm.	2.												—
- lepidula (Schw.)	3.								—	—	—	—	
Trifarina bradyi Cushm.	3,4.									—	—	—	—
Gyroidina soldanii d'Orb.	1.	x	x	x					—	—	—	—	—
- sp.	1.												
Eponides berthelotianus (d'Orb.) var. subornatus (Cushm.)	3,4.												—
- minima Cushm.	1,2.							—	—				
- praecinctus (Karrer)	1,2,4.	x	x	x	x	x					—	—	—
- umbonatus (Reuss)	3.								—	—	—	—	—
- - - var. multiseptus Koch	3.		x						—	—	—	—	
- sp. ?praecinctus var.	4.												
Rotalia beccarii (Linn.)	3,4.			x		x			—	—	—	—	—
- schroeteriana P.& J.	1.					x							—
Epistomina elegans (d'Orb.)	3.							—	—	—	—	—	—
Cancris auricula (F.& M.)	1.				x				—	—	—	—	—
Ceratobulimina eximia (Rzehak)	1.								—				
Ehrenbergina bradyi Cushm.	3.										—	—	—
Pullenia sphaeroides (d'Orb.)	1,3.	x						—	—	—	—	—	—
Sphaeroidina bulloides	1,4.		x					—	—	—	—	—	—
Globigerina conglomerata Schw.	1,2,4.								—	—	—	—	—
- eggeri Rhumbler	4.												—
- aequilateralis Brady	1.	x		x	x			—	—	—	—	—	—
- ?fistulosa Schubert	4.											—	
- quadripartita Koch	1,2.		x	x						?--	—	--?	
- triloba Reuss	1,2,3,4.	x	x	x		x		—	—	—	—	—	—
- tripartita Koch	1,2,4.		x	x						?--	—	--?	
- sp. Koch 1923	1,2.	x								?--	—	--?	
Globigerinella subcretacea Lomnicki	3.												—
Hastigerina pelagica (d'Orb.)	4.					x					—	—	—
Orbulina universa d'Orb.	1,2,3,4.	x	x	x	x	x		—	—	—	—	—	—
Pulleniatina obliquiloculata (P.& J.)	4.	x		x		x				—	—	—	—
Sphaeroidinella dehiscens (P.& J.)	4.												—
Globorotalia crassula Cushm. & Stewart	3,4.								—	—	—	—	
- menardii (d'Orb.)	1,2,3,4.	x		x		x		—	—	—	—	—	—
- tumida (Brady)	1,2,4.	x		x		x			—	—	—	—	—
- - var. flexuosa Koch	1.	x		x						?--	—	—	
Anomalina alazanensis Nuttall	1,3									—			
- cf. evoluta Le Roy	3,4.				x						—		
- flintii Cushm. var. ngimbangensis v.d.Sluis & de Vletter n. var.	1.												
- ornata (Costa)	1.										—	—	—
- sp.	4.												
Planulina bradyi Tolm	2,3.										—	—	—
- fijiensis Cushm.	1,3.											—	
- wuellerstorffi (Schw.) var. 1	1,2,3,4.												
- - - 2	2.												
Laticarinina pauperata P.& J.	3.	x	x		x				—	—	—	—	—
Cibicides americanus Cushm.	2.	x							—	—	—		
- javanensis v.d.Sluis & de Vletter n. sp.	2.												
- mantaensis (Gall. & Morr.)	1.								—	—			
- ungeriana (d'Orb.)	1,2,4.	x				x		—	—	—	—	—	—
- sp. aff. C. formosa Seguenza	4.												
Planorbulinella larvata (P.& J.)	1.	x	x	x	x				—	—	—	—	—

Greatest diameter: 0,63 mm, greatest thickness: 0,22 mm.

Bolivina dilatata REUSS var. *javana* n. var. fig. 4*a*, *b*. 113 ×.

Test nearly 2½ times as long as broad. The variety has less chambers (13—15), which are more inflated and higher; sutures more limbate and depressed; aperture broader.

Length: 0,4 mm, breadth: 0,15 mm.

Bolivina scalprata SCHWAGER var. *miocenica Macfadyen*. fig. 5*a*, *b*, *c*, *d*. 115 ×.

We got a large number of specimens which in our opinion belong to *Bolivina scalprata* SCHW. var. *miocenica* MACFADYEN. One part of our specimens (fig. 5*c*, *d*) shows much resemblance with the original picture of MACFADYEN, but the other part (fig. 5*a*, *b*) is somewhat different: their size is small (up to 0.44, but more often about 0.30 mm) and the specimens are less quickly tapering and have therefore a more rhomboidical shape. These smaller specimens are possibly the microspheric forms. None possesses any costae, which they ought to have according to CUSHMAN's description but not according to his picture in his monograph of the *Virgulininae*. Our specimens show a slight longitudinal thickening in the middle of the median planes. Lastly the excavations of the chambers of our smaller specimens are mostly deeper than indicated in the figures of CUSHMAN.

Gyroidina sp. fig. 6*a*, *b*, *c*. 43 ×.

Biconcave, conical, the dorsal side slightly concave, ventral with a deep umbilicus, last whorl with 11 chambers, sutures slightly elevated, limbate, curved backward; thick spiral sutures; coarsely perforated; chambers growing in size as added; aperture a narrow slit at the inner margin of the last chamber.

Greatest diameter: 1.20 mm. Greatest height: 0.74 mm.

Eponides sp.

This species is very much alike *Eponides praecinctus* (KARRER) only differing in having a knob on the ventral side. It may be a variety of *E. praecinctus*, but we have insufficient material to be sure of it.

Anomalina flintii CUSHM. var. *ngimbangensis* n. var.

Variety differing from *Anomalina flintii* CUSHM. as follows: test more compressed, sutures more curved, chambers becoming higher as added.

Anomalina sp. fig. 7*a*, *b*, *c*. 42 ×.

Test compressed; periphery lobulate; subacute to rounded; evolute; on both sides showing two whorls; the ventral side concave with a deep umbilicus; the dorsal side slightly convex; aperture an arched slit at the base of the last chamber and ventrally extending for the half of the last whorl; 8 chambers in the last whorl; sutures depressed, slightly curved on the ventral side; wall coarsely perforated.

Greatest diameter: 0.82 mm. Greatest height: 0.33 mm.

Resemblance with *Anomalina dorri* COLE.

Planulina wuellerstorffi (SCHW.) var. 1.

Variety differing in having a knob at the centre of the ventral side.

Planulina wuellerstorffi (SCHW.) var. 2.

This variety resembles much *Planulina wuellerstorffi*, differing only slightly in form; possibly it is a pathological case. It resembles very much a normal form of the abnormal *Pl. wuellerstorffi* as described and drawn by KOCH in the Eclogae geologicae Helvetiae, Bd. XIX, no. 3, 1926, p. 748, fig. 24*a*, *b*.

Cibicides javanensis, n. sp. fig. 8*a*, *b*, *c*. 40 ×.

Test biconvex, ventral side sometimes flat, dorsal side sometimes strongly convex; periphery acute; slightly lobulated and with a rim of clear shellmaterial; chambers distinct, 7 to 8 in the last whorl; sutures on the ventral side radial, on the dorsal side strongly oblique; aperture a narrow slit at the inner margin of the last chamber, extending a short distance on the dorsal side.

Greatest diameter: 0.55 mm. Greatest height: 0.28 mm.

Not a single *Cibicides* shows resemblance with this species. The general form of *Eponides karsteni* (REUSS) shows the most resemblance.

The faunae, referred to in the list, are described in the following publications:

BOOMGAART, L. and VROMAN, J.: Smaller Foraminifera from the Marl Zone between Sonde and Modjokerto (Java). Kon. Akad. Wetensch. Proc., Vol. XXXIX, no. 3, 1936.

FISCHER, P. J.: Eine Pliocänfauna von Seran (Molukken). Centralbl. f. Min., Geol. u. Pal., no. 19, 1921.

KOCH, R.: Die jungtertiäre Foraminiferenfauna von Kabu (Res. Surabaja). Eclog. Geol. Helv. Bd. XVIII, no. 2, 1923.

KOCH, R.: Eine jungtertiäre Foraminiferenfauna aus Ost Seran (Kasuma). Eclog. Geol. Helv. Bd. XIX, no. 1, 1925.

KOCH, R.: Mitteltertiäre Foraminiferen aus Bulongan, Ost Borneo. Eclog. Geol. Helv. Bd. XIX, no. 3, 1926.

ROY, L. W. LE: Some small Foraminifera, Ostracoda and Otoliths from the Neogene ("Miocene") of the Rokan-Tapanoeli area, Central Sumatra. Natuurk. Tijdschr. voor Nederl. Ind. XCIX, Nov. 1939.

YABE, H. and ASANO, K.: Contributions to the Palaeontology of the Tertiary formations of West-Java. Part I-Minute Foraminifera from the Neogene of West Java. Sci. Rep. Tohoku Imp. University. 2nd Ser. (Geol.) Vol. XIX, no. 1, 1937.

Palaeontology. — *On a new genus of arenaceous foraminifera from the Cretaceous of Texas.* By F. KEIJZER. (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of November 28, 1942.)

In 1934 the Geological Institute of the University of Utrecht received from Dr. P. M. ROGGEVEEN, together with a collection of fossils from Texas, a slide containing a number of foraminifera, and labelled: "Texas, W. of Austin, surface sample Walnut". The latter specimens could not be specifically identified, or even be placed in an existing genus. They are described here as belonging to a new genus:

Coskinolinoides nov. gen.

Genotype: *Coskinolinoides texanus* n. sp.

Test conical, arenaceous, many chambered. Early portion a trochoid spire, starting with 4 chambers in a whorl, later becoming uniserial. Uniserial chambers, and possibly also later chambers of spiral portion subdivided by radial septa. Aperture of the spiral portion at the inner margin of the last-formed chamber. Aperture of uniserial portion consisting of rounded pores in the central area of the roof of the last-formed chamber.

Occurrence: Lower Cretaceous (Walnut formation = lower middle Albian), Texas, U.S.A.; ?Lowermost Cretaceous (Valanginian), Southern France.

According to its plan of growth, the genus belongs in the family of the *Valvulinidae* (*2*, p. 114; *3*). Within this family it shows affinities with the group *Lituonella-Coskinolina-Dictyoconus* in the general shape of the test, radially partitioned uniserial chambers and apertural characters. Moreover it shows a slight tendency toward developing a marginal trough and perforated central shield (compare fig. *d* with *3*, p. 171, f. 1), but no evidence could be found of the presence of vertical pillars between the floors and roofs of the uniserial chambers. The absence of pillars is the only characteristic, which definitely separates it from *Coskinolina* (*3*, p. 186). Other differences of the genotype of *Coskinolinoides* with species of *Coskinolina* are only gradual and may have no generical value, e.g. the smaller size of the specimens, and the smaller number and relatively greater length of the radial septa in the uniserial chambers.

Superficial resemblance exists with *Liebusella* (3, p. 162), but the latter differs in the shape of the uniserial chambers and apertural characters.

Small, conical, arenaceous foraminifera, described and figured by PFENDER (*5*, p. 237, pl. XIV, f. 6, pl. XV, f. 4) may possibly also be referred to *Coskinolinoides*. These occur in Southern France in limestones of lowermost cretaceous age, together with *Dictyoconus walnutensis* (CARSEY). They are refigured here (fig. *i*, *j*).

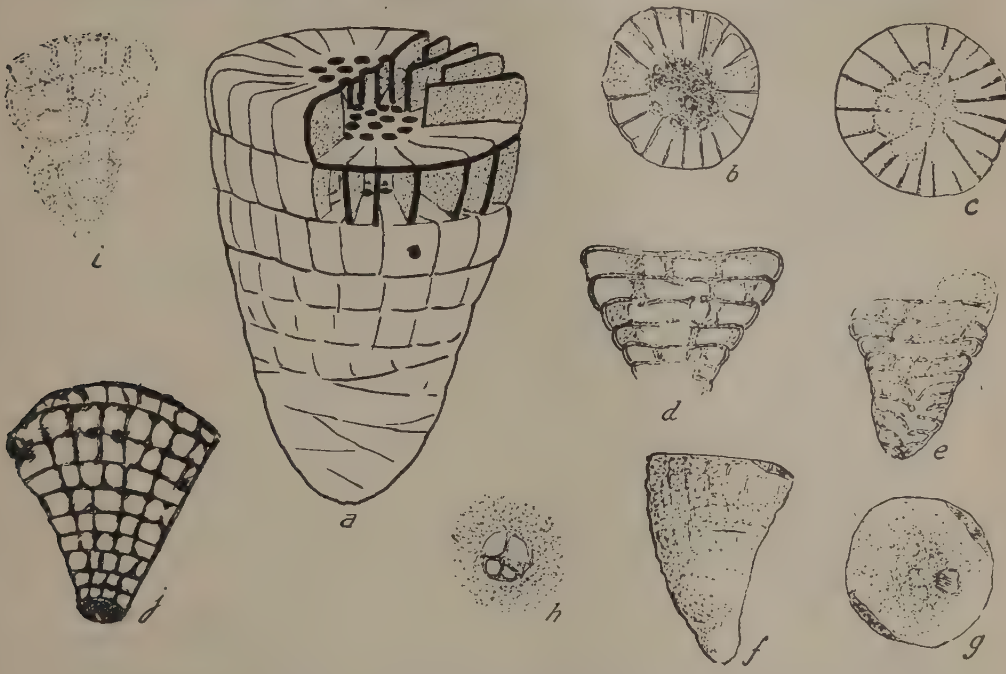
Mention of small, conical foraminifera from the lower Cretaceous of Louisiana is made by CUSHMAN (*2*, p. 174, as *Orbitolina*) and by GALLOWAY (*4*, p. 204, as *Coskinolina*), but the author failed to find any description of such forms. They have not been listed by VAUGHAN (*7*). Neither could any clue be found in the handbooks on the paleontology and geology of Texas (*1*,*6*). This was rather surprising, as our specimens were in one collection with slides, containing picked faunal assemblages, composed of well-known characteristic species, from other cretaceous formations of Texas.

Coskinolinoides texanus n. sp.

Since the genus is based on a single species, the description would merely recapitulate that given for the genus, and only the following data of specific importance are added here:

Test small, conical, apex often somewhat bent to one side. Height up to 0.43 mm; the diameter at the base may equal the height, but is generally a little less. Number of uniserial chambers up to 6; height of uniserial chambers about 55 μ; diameter of apertural pores about 15 μ; thickness of wall, septa, roofs and floors 10 μ.

While no section gave conclusive evidence about the earlier chamber-arrangement, one specimen with a broken apex showed the early chambers forming a whorl of 4 chambers.



Explanation of figures: *a*: Schematized reconstruction of *Coskinolinoides*, showing structure of younger chambers; *b—h*: *Coskinolinoides texanus* (*b*, *c*: horizontal sections; *d*, *e*: vertical sections; *f*: side view; *g*: apertural view; *h*: broken apex, showing arrangement of early chambers; *b—g* × 65); *i*, *j*: ? *Coskinolinoides* sp. sp., redrawn after PFENDER l.c. 1938 (*i* × 120; *j* × 40).

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Histology. — *The influence of Hormones on the Os Priapi.* By S. T. Bok, S. E. de Jongh and I. van Zwanenberg. (From the Histological and the Pharmacological Laboratories of the University of Leiden. Directors: Prof. Dr. S. T. Bok and Prof. Dr. S. E. de Jongh.) (Communicated by Prof. J. van der Hoeve.)

(Communicated at the meeting of November 28, 1942.)

In a previous paper [1]) we pointed to the following facts: Testosterone, when administered for 4 weeks to castrate immature rats induces a strong development of the os priapi. In the hypophysectomized castrate testosterone causes an enlargement, comparable with the foregoing, but the differentiation is less distinct. A difference exists between hypophysectomized and intact *un*treated castrates in view of loss, resp. persistance of the cartilage capsule on the proximal end.

Two questions remained still unanswered:

1. Is the difference, in view of the development of the os priapi between hypophysectomized and intact castrates, treated with testosterone, essential or is it but a question of tempo?

2. Is the influence, that appears to be exerted by the hypophysis, due to growth hormone?

We tried to solve the former question by enriching the already available material with non-hypophysectiomized castrates, treated with testosterone (in stead of for 4 weeks!) resp. during 1, 2 and 3 weeks. We then investigated, if we could find in the thus obtained series a place for the hypophysectomized animals treated for 4 weeks.

The second question gave rise to experiments, in which hypophysectomized castrates received, besides testosterone, growth hormone, in order to realize an, albeit perhaps partial, substitution of the hypophysis.

We are indepted to Prof. Dr. E. Laqueur and Dr. Elizabeth Dingemanse, who suplied us with growth hormone, a rather pure preparation, containing 1 R U per 50 γ.

The new experimental material is recorded in table I and II. The histological description as well as the comparison with the already available microscopical preparations (see previous article) gave rise to the following notations.

Normal architecture.

In young rats (as had been used in these experiments) the penis skeleton consists in 1. the os priapi and 2. a solid body, situated in the middle of the glans, distally and somewhat dorsally of the os priapi.

The os priapi has the shape of a round-headed nail: its longest part, the shaft, the axis of which runs parallel to the urethra, has a fairly constant width, its proximal end is broadened: the head. In the shaft the medullary cavity is narrow, its diameter measuring about 0.1 of the total diameter of the shaft, in the head it is wide, the wall (compacta) of the head being scarcely thicker than that of the shaft. The cavity contains wide mashed reticular connective tissue, no bone rods (trabeculae) are passing through it, in the head its surface (the endost) is slightly irregular. Within the compact bony wall of the head some strips of collagen connective tissue are found and the superficial layer of its most proximal part consists of hyalin cartilage. This cartilage thus forms a dome shaped cartilage cupule at the proximal pole of the os priapi. It immediately adjoins the osseous tissue.

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[1]) Acta. N. Morph. 2, 236, 1939.

The general bone lamellae being fairly distinct, in the cross sections especially the shaft shows a pattern like that of the annual rings in trees.

The solid body of the glans penis consists of fibrillar cartilage containing a dense complex of collagen fibres. It is surrounded by a distinct layer of connective tissue.

TABLE I.

Castrated young male rats, treated with testosterone.

Number of rats	Animal weight (extreme values)		Treatment	
	beginning	end	substance and dosage	during
4	99—132 g	121—169 g	300 γ testosterone-propionate	7 days
4	89—128 g	158—188 g	300 ,, ,,	14 ,,
4	94—118 g	164—192 g	300 ,, ,,	21 ,,

TABLE II.

Castrated and hypophysectomized young male rats, treated with testosterone and/or growthhormone. Castration 4 weeks before, treatment 1 week after hypophysectomy.

Number of rats	Animal weight (extreme values)		Treatment	
	beginning	end	substance and dosage	during
3	100—140 g	101—147 g	300 γ testost. prop.	28 days
3	87—131 g	108—138 g	10 U. growthhormone	28 ,,
5	101—132 g	138—154 g	test. prop. + growthhormone	28 ,,
4	89—126 g	86—122 g	olive oil[1]) and NaOH[2]) 0.01 N	28 ,,

Structure of the penis skeleton in the castrates.

In the cross sections the os priapi and the glans body appear as to be much smaller than normal: the diameter of the head is about 50 % and that of the shaft about 70 % of those diameters in normal animals of the same age. Only some of the sections being mounted, no information about the longitudinal dimensions is available. The difference between the degrees of hypoplasia in head and shaft are due to the fact that the diameter of the medullar cavity is more below the standard than the thickness of the bony wall. The internal (endostal) surface of the headwall is less irregular (more flattened) than normal.

The glans body of a castrate does not contain any cartilage cell: it thus exclusively consists of dense collective tissue.

Hypophysectomised castrates.

In the castrates, the hypophyseal gland of which has been removed, the hypoplasia of the penis skeleton is more marked yet. Especially the compact wall is very thin. Moreover its finer structure is somewhat irregular as if some shrinking has occurred. In the head the cartilage cupule is absent.

Influence of control injections.

As a control experiment, oil was injected in some of the animals. They did not show another microscopical picture than those without oil injections.

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[1]) Solvent of testosterone propionate.

[2]) Solvent of growthhormone.

The influence of testosterone injected in castrates, the hypophyseal gland of which was not removed.

When testosterone is injected repeatedly into a castrate, the hypophyseal gland of which was not removed, the os priapi and the glans body are found to be larger than they would have been without these injections. After one week the diameter of the head is already twice as large as in an animal spayed at the same age and sacrificed after the same interval. During the following weeks this effect continues but decreasing in strength: the enlargement of the shaft and of the cartilage cupule stop after two weeks of treatment and in the 3d and 4th week the enlargements of the head and of the glans body are small.

During the first week the enlargement of the head is caused exclusively by a development of cartilage; at the end of that week the osseous tissue and the medullary cavity show the same conditions as in an untreated castrate, but the external surface of the bone is fully surrounded by a layer of new formed hyaline cartilage 2—3 times as thick as the original bone wall.

Then connective tissue, probably originating from the strips inside the bony wall, penetrate into the inner parts of the new formed cartilage layer. Very soon this connective tissue ossificates, so that after 2 weeks of treatment a thick layer of new formed bone is found all around the old bone tissue. The young bone is arranged in irregular rods; between them fields of connective tissue and of hyaline cartilage are scattered about. The bony rods tend to a radial direction and they grow towards the perifery. Their periferal parts tend to a mutal contact and at some spots they already reach the surface of the head.

After these two weeks the medullary cavity of the head is slightly enlarged: the endostal resorption of the original bone has begun at some spots, causing distinct irregularities of the internal (endostal) surface of the bony wall.

During the 3d and 4th week these processes continue, the growth of cartilage tissue at a rapidly decreasing rate and the ossification at a slowly decreasing rate while the endostal resorption at first increases in intensity and then decreases also. After 4 weeks of treatment the medullary cavity of the head is approximately as large as in the normal animals of the same age, the wall is somewhat thicker. Within that wall the cartilage tissue is building up an abnormally large proximal cupule with offshoots along the lateral surfaces. Within the bone tissue abnormally large and numeral fields of connective tissue are present, especially in the lateral parts of the head wall.

The periost of the head of the os priapi hypertrophies as well, especially during the 3d week.

The shaft of the priapis bone answers the testosterone injections by periostal growth of the bone. During the 3d week it reaches the thickness of a normal non-castrated animal of the same size. A distinct "annual ring" indicates the circumference of the original bone: obviously the shaft did not grow during the period (5 weeks) between the castration and the administration of testosterone. In the shaft the width of the medullary cavity does not alter during the testosterone treatment: here no endostal resorption occurs.

The glans body not only grows during the testosterone treatment — it reaches normal dimensions in 2 weeks — it shows a histological differentiation as well: in a castrate it being built up by connective tissue exclusively, cartilage cells appear in it as soon as testosterone is injected; the number of these cells increases during all the 4 weeks of the experiments and after these 4 weeks they are rather more than less numerous than normally.

Influence of testosterone upon castrates, the hypophyseal gland of which was removed.

After 4 weeks of treatment with testosterone injections, the hypophysectomized castrates show a microscopical picture that closely resembles that which may be expected in non-hypophysectomized castrates after 1½ week of testosterone injections.

The external dimensions of the penis skeleton are between those of the latter with 1 and with 2 weeks treatment, the medullary cavity still has the extension as in the untreated castrate of the same age. The strips of connective tissue, entering the bone from the endost, are somewhat hypertrophied. In the central parts of the new formed periferal cartilage layer some small bone rods have developed (in the non-hypophysectomized castrates this cartilage after 1 week of treatment does not yet contain any of such osseous rods and after 2 weeks many of them).

In many of these animals the growth of the head is asymmetrical. In the parts where the growth is less no cartilage is found; in stead of it between the periost and the original bone a thin layer of new formed bone can be found, which in all probability has been built by periostal ossification.

The shaft reveals the same picture as in a non-hypophysectomized castrate after 2 weeks of testosterone application.

The dimensions of the solid body in the glans, in contrary, correspond to those of the same stages in castrates possessing a hypophyseal gland: after 4 weeks treatment the number of cartilage cells is very large, even larger perhaps than in the non-hypophysectomized castrates with 4 weeks testosterone injection, and these cells themselves are larger.

Influence of Growth Hormone upon Hypophysectomized Castrates.

The administration of growth hormone for 4 weeks caused no more changes in the castrate than the development of a cartilage cupule in the wall of the head of the os priapi in 2 of 3 animals.

Influence of combined administration of Testosterone and Growth Hormone in the Hypophysectomized Castrate.

In the hypophysectomized castrate the combined injection of testosterone and growth hormone for 4 weeks produced a picture that practically corresponds with that in the non-hypophysectomized castrate after 2 weeks testosterone administration: the head is larger than after testosterone alone; the original medullar cavity is still recognizable, here and there the wall is arroded and considerable, rather irregularly out-lined processes are present. The major part of the head's wall consists of bone rods with much collagen connective tissue between them; the outer layer still consists of cartilage tissue. The size as well as the ossification of the cartilage and the endostal resorption of the original bone are approximatively at the same stage as in a non-hypophysectomized castrate after 2 weeks testosterone administration and distinctly further developed than in hypophysectomized, spayed recipients after 4 weeks exclusively testosterone.

After the combined administration of growth hormons and testosterone the shaft of the os priapi and the solid body in the glans of the hypophysectomized castrate show almost the same picture as after the application of testosterone alone for the same period.

Discussion.

The reaction of the penis skeleton upon the different injections described above has two aspects: a growth of the skeleton and structural changes.

The growth of the os priapi is due to an activity of the periost: in the head the periost builds cartilage, in the shaft it builds bone. The new formed cartilage has a much larger volume than the bone, formed in the same time. In consequence the head grows much more than the shaft.

Soon after the cartilage has developed, its ossification begins: from the strips of connective tissue lying within the osseous tissue of the head streams of connective tissue penetrate into the central parts of the cartilage and some days later the larger part of this new built connective tissue transists into bone. The young bone rods tend to a radial arrangement, they grow towards the perifery and there they grow towards each other. About a week after the beginning of the ossification the endost starts to resorb the central parts of the bony wall, thereby enlarging the medullary cavity.

In the shaft, where no ossification of cartilage occurs, the endostal bone resorption is absent.

When the growth stimulus is very week some parts of the periost in the head form bone in stead of cartilage. At these spots the growth is less than in the adjacent parts building cartilage.

The growth of the glans body is due to a hypertrophy of the connective tissue. Meanwhile cartilage cells develop within that tissue.

From the foregoing description it is obvious that in these reactions testosterone is the major stimulus (at least in the given dosages) in stimulating growth and producing structural changes and that the pituitary component acts mainly by *accelerating* these processes.

It is very probable, that this component may be identified with growth hormone:

1. The reappearance of the cartilagenous capsula on the head of the os priapi in the hypophysectomized castrate, receiving exclusively growth hormone.
2. The combined administration of testosterone and growth hormone to the hypophysectomized castrate produced a picture, resembling that of the non-hypophysectomized recipient, treated with testosterone for 2 weeks and much further developed than that of the hypophysectomized castrate, that received solely testosterone for 4 weeks.

Our experiments do not answer the question wether or not it is possible, to obtain the same results with a *chronic* application of testosterone alone as with the administration of testosterone to the non-hypophysectomized recipient for 4 weeks.

Conclusion.

The os priapi and the solid body in the glans are smaller and of other architecture in the castrate than in the normal animal of the same age.

The administration of testosterone produces a practically complete repair of size and structure. The course of this can be studied in 4 phases.

The administration of testosterone induces the same process of augmentation and differentiation in hypophysectomized as in non-hypophysectomized castrates, but at a lower speed. The rapidity of this process increases, when growth hormone is given at the same time, so this substance is capable of partially replacing the pituitary gland.

The nature of the process is not changed by the concomitant administration of growth hormone, as far as can be deduced from the histological preparations.

In the untreated castrate hypophysectomy causes only slight further changes; the latter are mainly expressed in the disappearance of the cartilage capsula in the head of the os priapi. The administration of growth hormone caused the reappearance of this structure.

Anatomy. — *Placodes of cranial ganglia in Reptiles.* By J. ARIËNS KAPPERS. (Communicated by Prof. M. W. WOERDEMAN.)

(Communicated at the meeting of November 28, 1942.)

Introduction.

Placodes are circumscript areas of ectoderm which participate in the formation of sense organs, cranial ganglia and to all probability in the formation of the supporting tissues in the head, after the beginning of differentiation of the ectodermic areas of the neural plate and the neural crest. The development and function of the placodes, which participate in the formation of the cranial ganglia, to which we will confine us in this paper, are well studied in Fishes and especially very extensively, also experimentally, in Amphibia; much more scarce, however, and less agreeing are the data about Sauropsida and Mammalia. Therefore a renewed investigation concerning the placodes in Reptilia seems justified.

From the earlier authors who at least have mentioned or figured the placodes, here to be discussed, HOFFMANN (1886) is the first. Although deriving both components of the trigeminus ganglion wholly from the neural crest, he describes and figures in *Lacerta* and *Tropidonotus* a "cutaneous branch" which takes its rise from the ganglion ophthalmicum. This "branch" has contact with the epiderm and, as it seems, receives cells from it. It is highly probable that HOFFMANN in this way describes the ophthalmicus placode without knowing its real interpretation. Furthermore, VAN BEMMELEN (1888) must be mentioned. He observed how in *Lacerta* the "trunk" of the ganglia of the VIIth, IXth and Xth nerve gets into contact with the most dorsal recess of the respective branchial clefts and also figures a reconstruction of a *Lacerta* embryo from which it appears that these ganglia are connected with the dorso-caudal wall of the branchial clefts. OSTROUMOFF (1888a) describes in *Phrynocephalus* the placodes connected with the first, the second and the third branchial cleft. According to him the respective ganglia are closely connected with the thickened ectoderm. He looks upon these placodes as branchial sense organs, following the theory of BEARD and FRORIEP. In his more extensive work (1888b), only published in the russian language, he gives also some figures in which these placodes, evidently epibranchial ones, occur. On the other hand OSTROUMOFF states that the V ganglion is not connected with the epiderm in the way of the branchial sense organs. BEARD (1888) only states that in the lizard, *Triton* and rabbit the development of the cranial ganglia is essentially the same as that described by him in sharks "with the slight exception that the lateral or branchial sense organ Anlagen are more or less rudimentary and embryonic, or larval, structures". In 1904 SAINT-RÉMY and PRENANT, giving an elaborate description of the genesis of the branchiogenic organs in Sauria and Ophidia, also mention the epibranchial placodes of the hyomandibular clefts and figure them. Apparently other placodes were not seen by these authors.

Among the more recent workers in this field BRACHET (1914) confined himself to a close examination of the formation of the trigeminus ganglion in *Chrysemys*. Here he found a large trigeminal placode, existing of two parts, an ophthalmic and a maxillo-mandibular one. The latter should only contribute in a minor degree to the formation of the ganglion. The ophthalmic component, however, is for the larger part constituted out of the respective placode, which is also more sharply circumscribed than the maxillo-mandibular placode according to this author. FRADE (1938) described in *Sternothoerus*, another turtle, also only trigeminal placodes. The ophthalmic placode and the maxillo-mandibular one, which occurs somewhat later, present themselves as rather deep epidermic grooves out of the bottom of which cells migrate which get into contact with the cells of the neural crest. According to FRADE both placodes give a substantial contribution to the respective parts of the trigeminal ganglion. The most extensive paper on this subject is that of EVANS (1934), who described in detail in a series of developmental stages of *Gymnodactylus* embryos the two trigeminal placodes, epibranchial placodes connected with the VII, IX and X ganglia and even dorso-lateral placodes which he found in some very rare cases. In our paper we will have the opportunity to quote more than once the results of his investigations.

Material.

My investigation is concerned with 28 sorted series of embryonic Reptiles, mostly stained with haematoxyline-eosine; only very few were stained with eosine only. The following genera were represented (the figures between the brackets referring to the number of series used): *Chelydra* (3), belonging to the Testitudinata; *Hemidactylus* (4), *Mabuia* (3), *Sept* (2) and *Lacerta* (6) belonging to the Lacertilia; *Coluber* (8) and *Tropidonotus* (2) belonging to the Ophidia. Thus among the Reptilia only representatives of the Crocodilia and of the Rhynchocephala were lacking.

These series were kindly placed at my disposal by the courtesy of Prof. WOERDEMAN from the Anatomical Laboratory at Amsterdam, Prof. VAN DEN BROEK from the Anatomical Laboratory at Utrecht, Dr. DE LANGE from the Embryological HUBRECHT Laboratory at Utrecht and Dr. DANKMEYER from the Anatomical Laboratory at Leiden. I am greatly indebted to these gentlemen for the opportunity to make use of this valuable material. The figures I owe to the skill of Mr. KERSSEN, of the Anatomical Laboratory at Groningen.

It appears that the relations in the various genera are very similar. Therefore, and also because of the relatively small material of each genus, the placodes in Reptiles will be dealt with in a rather generalised way, that is, the ontogenetic history of placodes in Reptiles will be reconstructed from the available material. We can only hope that we are later enabled to test our statements on a more abundant material. First the epibranchial placodes and their elements will be described, then the occurrence of dorso-lateral placodes will shortly be dealt with and at last the trigeminal placodes will come to discussion. Also the question will be answered whether mesectoderm, other than from the neural crest, is formed, as is the case, also proved experimentally, in Ichthyopsida.

Description.

Epibranchial placodes. Our youngest specimen was a *Lacerta* embryo in which 10 somites were counted. The otic placode is scarcely invaginated, the neural folds are still widely open and the first branchial pouch just reaches the ectoderm. In this embryo the ectoderm of the whole head not only relatively but also absolutely is much thicker than in older stages. In the fore part of the head the ectoderm consists of cubic or prismatic cells and has only one layer of cells; more caudally, however, in the later branchial region, the ectoderm contains more layers except for the dorsal region, where there is only one, the cells of which show a cubic or even a flattened form. The entire thickening is confined to the head (Fig. 1). In a somewhat older *Coluber* embryo (Fig. 2) the otic placode is already deeply invaginated and the neural tube quite closed but the membrana obturans of the hyomandibular pouch is not yet ruptured. There is as yet no sign of lens formation. In this embryo the primary ectodermic thickening in the fore part of the head has already vanished by flattening of the cells. Only in the branchial region the ectoderm is still much thickened and on the whole consists of two or three layers of cells. This area of thickened ectoderm is ventrally bounded by the deep groove which sets the heart apart from the rest of the embryo. Dorsally this area includes the epibranchial as well as the dorso-lateral ectoderm. From the latter the otic placode is formed and in Ichthyopsida also the primordia of the lateral line placodes. More dorsally, the ectoderm consists now exclusively of flattened cells. Cranially this general branchial thickening extends to the area of contact between the hyomandibular pouch and the ectoderm and caudally to the hind border of the head, *i.e.* to a level a little behind the evagination of the last visceral pouch. In later stages this general branchial thickening disappears and only localised ectodermal thickenings persist, which are found where the visceral pouches get into contact with the ectoderm. They are situated at the dorso-caudal walls of the visceral pouches, or, afterwards, of the branchial clefts, and consist of more than one layer of cells (Figs. 3 and 4). These ectodermal thickenings become placodes by proliferating cells in a mesio-dorsal direction. The situation, just described, struck all former authors who ever described reptilian epibranchial placodes. This is also always the localisation of epibranchial placodes in Ichthyopsida.

The migration of cells out of the placodes occurs singly as well as *en masse*. They move along a shorter or longer distance in a mesio-dorsal direction to that group of cells which, having its origin in the neutral crest, takes also part in the formation of the respective ganglion (Fig. 5). The migrating cells may form a long row or procession (Fig. 6). The migration of cells from the epibranchial VII placode begins about the time at which the membrana obturans of the hyomandibular pouch ruptures. The same is *c.p.* true for the epibranchial IX placode. Afterwards, the proliferation of the epibranchial X placodes begins. The first three epibranchial placodes (VII, IX and X1) were always easily observable; the second epibranchial X placode, however, not in all cases and the third only in one embryo, *viz.* a *Chelydra*. In this connection it must be kept in mind that we only call an ectodermal thickening a placode if it proliferates. The epibranchial VII placode starts proliferation, then the IX placode and afterwards the X placodes follow. However, it is not true that the VII placode stops proliferating when the IX placode begins etc. There is a time when all epibranchial placodes are more or less engaged in shedding cells. That the VII placode represents itself as "a grooved structure" as EVANS describes in the Gecko, does not follow from our investigation.

Now we will draw attention to a fact which we consider as rather important and very conspicuous, but which is not mentioned by any author, although it is to be seen on some of the drawings which illustrate the paper of SAINT–RÉMY and PRENANT and of EVANS. The cells which migrate from the epibranchial placodes show some histological differences from the cells of the neural crest to which they are added. The latter have a nucleus which is somewhat smaller, their form may be round but may also be oval, they are less clear and contain many corpuscles. The protoplasma of these neural crest cells stains rather dark in later stages. The placodal cells, on the other hand, are always very well identifiable by their nucleus which is practically round, rather large and very clear. It contains a nucleolus which lies in the middle of the otherwise nearly empty nuclear plasma. The protoplasma of these placodal cells is not so darkly stained but much more abundant than that of the cells of neural crest origin. In young stages there is no difference in stain of protoplasma between placodal cells and neural crest cells. These histological differences remain during a very long time. Also after fusion of the epibranchial placodal cells with the neural crest group of cells they are still present so that then by these characters the various cells betray their origin. The placodal components of the ganglia with their cells having these large vesicular nuclei, form during a long time the ventro-antero-lateral part of the respective ganglion (Fig. 7) and in the oldest specimens under investigation we saw nerve fibres initiated from these parts running to the wall of the pharynx as well as into the branchial arches. In the Discussion the specific function of the placodal cells will be more fully dealt with.

Dorso (Acustico)—lateral placodes. About these placodes we can be short. Only in one *Chelydra* embryo and in one *Tropidonotus* embryo ectodermal thickenings which may represent rudimentary dorso-lateral placodes were found. In the *Chelydra* embryo the ectoderm just antero-ventrally from the otic vesicle is clearly thickened in a few succeeding sections and consists of more than one layer of cells. In this case a real migration of cells out of this thickening is not demonstrable but certainly probable because a part of the VII ganglion is extended towards this ectodermal thickening. The cells of this extension may be derived from the rudimentary placode. The epibranchial VII placode which lies just ventrally to the rudiment is connected with the latter so that both placodes gradually merge into each other. In this embryo the lens vesicle had loosened itself from the epiderm, the first two branchial clefts were open and the two following only closed by a thin membrana obturans.

A similar situation was found in a slightly younger *Tropidonotus* embryo. Here an area of thickened ectoderm was found just dorsally to the epibranchial IX placode. Also here the dorso-lateral thickening was not sharply separated from the proliferating epibranchial placode, although it may be that in this case both thickenings were disconnected by a very small area of undifferentiated ectoderm. Observation was rather difficult here. In this case the dorso-lateral thickening clearly proliferated. The migrating cells had quite the typical histological character of epibranchial placodal cells, mentioned before. For the interpretation of these facts we refer to the Discussion.

Trigeminal placodes. The youngest stage in which something could be learned about the development of the V ganglion with certainty, is given by a *Lacerta* embryo in which the lumen of the otic vesicle still has a slight communication with the outer world. Only a lens pit is present. The membrana obturans of the hyomandibular pouch has just ruptured, the second one has reached the ectoderm and the next pouch is seen as a slight evagination of the pharynx. The epibranchial VII and IX placodes have just started their proliferation. The ectoderm of the anterior part of the head consists of only one layer of flat cells. An ophthalmic ganglion in the making, is seen as a condensation of cells besides the prosencephalon. This ganglion rudiment seems to receive a contribution from a spot in the ectoderm which, however, is hardly thickened in comparison with the surrounding ectoderm. Only some few cells have detached themselves out of the ectodermal context to join the V^1 ganglion which, it seems, is formed in this animal chiefly from neural crest cells. Naturally it is not possible to say with absolute certainty if the cells, already present in the ganglion, are really derived from the neural crest and not from the ectodermal placode during earlier stages, but the relations found in a still somewhat younger embryo of *Lacerta*, support our view very much. Our observation can be directly compared with that of HOFFMANN as stated in the Introduction.

Quite another situation was found in *Seps*. In a specimen in which the lens still shows a lumen and is not yet quite loosened from the epidermis, the first two visceral pouches are open whereas the third one still shows a thin membrana obturans. In this embryo the epibranchial VII placode proliferates, the IX placode is less active but also cells depart from the X^1 and X^2 placodes. A large ophthalmic placode is present which in this genus seems to contribute a large part to the ganglion. The placode presents itself as a very much thickened place in the ectoderm, consisting of many layers of cells. It is mesially a little invaginated. From the bottom of the groove thus formed (Fig. 8), cells loosen themselves which will be added to the neural crest elements. The latter contribute also to the formation of the V^1 ganglion but, as it seems, only to a minor degree. These facts agree with those found by BRACHET (1914) in *Chrysemys* and FRADE (1938) in *Sternothoerus* and could be less clearly observed by EVANS (1934) in *Gymnodactylus*. In this same embryo also the maxillo-mandibular placode is extraordinary clearly distinguishable (Fig. 9). Not far cranially to the cranial wall of the oral cleft it shapes itself as an ectodermal thickening, existing of more layers of cells but not invaginated like the V^1 placode. Cells detach themselves *en masse* and in a cluster from the ectodermal context and are added to the neural crest component of the maxillo-mandibular ganglion. The V^2 ganglion remains separated from the V^1 ganglion by a vein during a long time. From all these facts it follows that the placodal contribution to the ganglion GASSERI seems here to be very large.

In a somewhat older series of *Seps* in which also the lens is still slightly connected with the epiderm, but *e.g.* the VII placode has stopped proliferating, the V^1 ganglion is already wholly formed and the V^1 placode has nearly entirely disappeared. It only

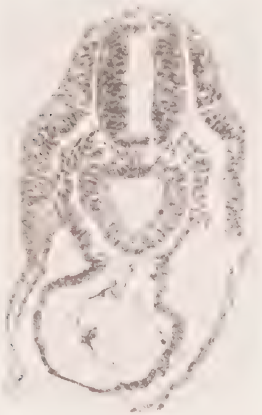

Fig. 1. *Lacerta ag.*, transv. sect. with otic placode and general branchial thickening.

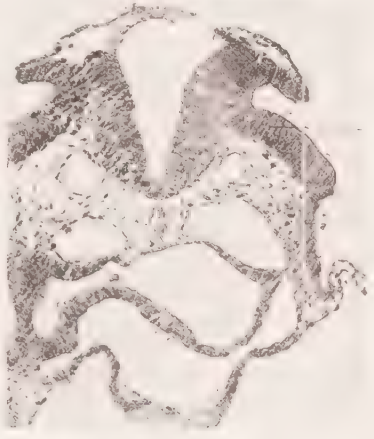

Fig. 2. *Coluber*, similar to Fig. 1 but a somewhat older stage; on the level of the second visceral pouch

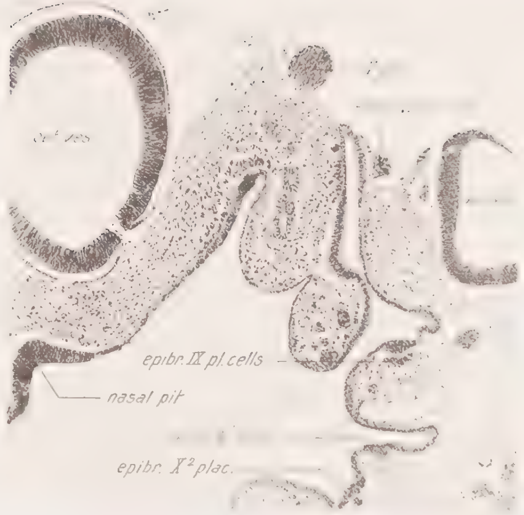

Fig. 4. *Hemidactylus*, sagitt. sect. The epibranchial placodes with their migrating cells for ganglia VII, IX and X are to be seen in one section.

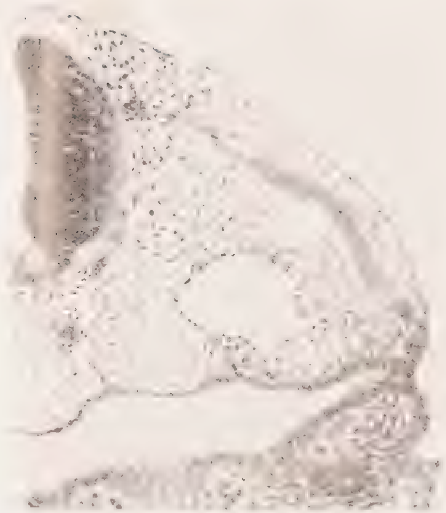

Fig. 6. *Seps*, transv. sect. Some neural crest cells and placodal cells for the X complex. The latter migrate from the epibranchial X¹ placode. Note the histological character of the placodal cells.

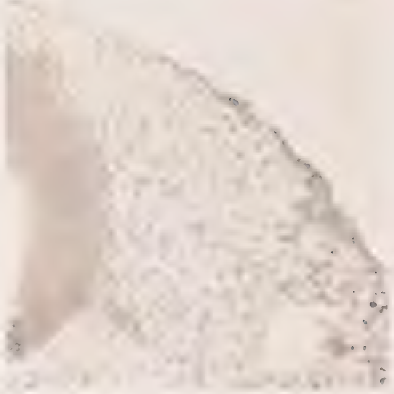

Fig. 8. *Seps* horizonto-transv. sect. Proliferating ophthalmicus placode. Note the N. III with Schwann cells.

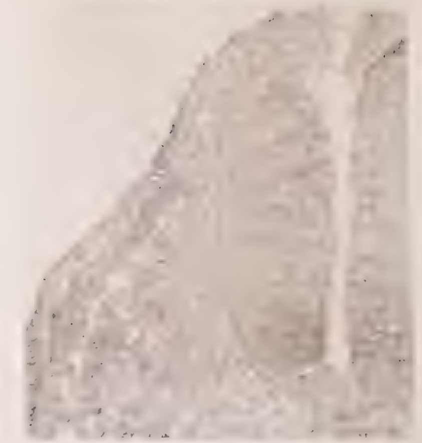

Fig. 10. *Mabuia*, transv. sect. Very young stage. The arrow points to a cell, migrating from the preotic dorso lateral ectoderm. The continuity of the basal membrane is broken.

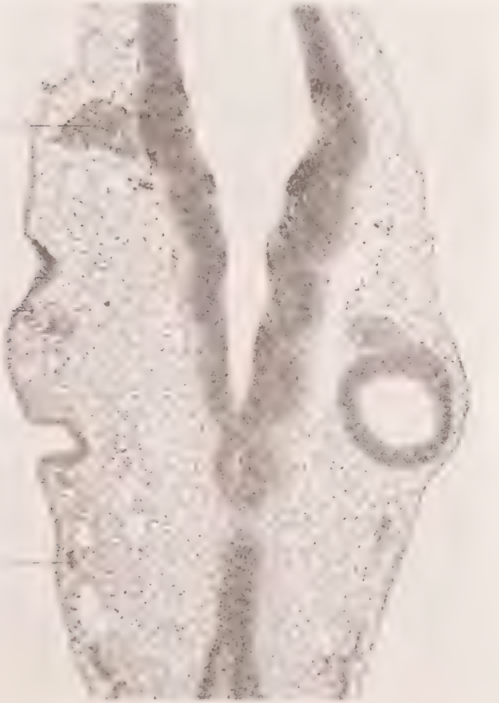

Fig. 3. *Seps*, horiz. sect. Epibranchial placodal cells for ganglia VII, IX and X. Note the topography of the VII and IX cells just dorso-caudally to the branchial clefts.

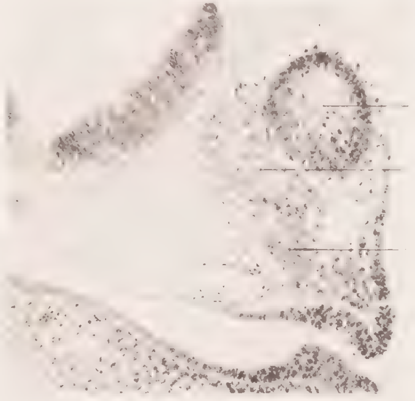

Fig. 5. *Seps*, transv. sect. Beneath the otic vesicle neural crest cells and placodal cells for the VII complex are visible. Note their difference in histological character. The ectoderm just above the hyomandibular cleft is thickened and still proliferates.

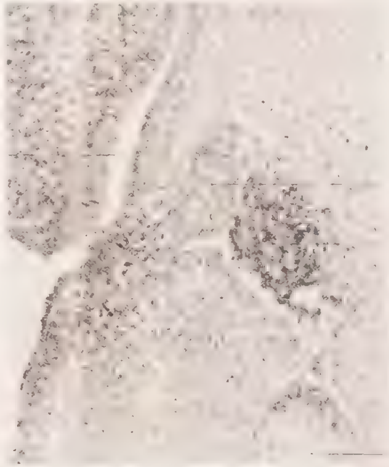

Fig. 7. *Coluber*, horiz. sect. Neural crest and placodal components of the VII ganglion in an advanced stage. Note the still existing difference in histological character and the ramus which leaves the placodal component caudo-laterally.

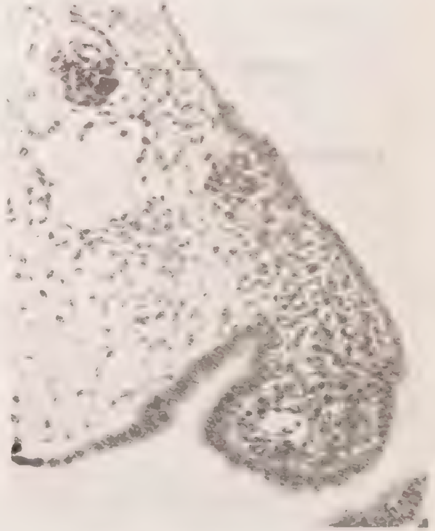

Fig. 9. *Seps*, horizonto-transv. sect. Maxillo-mandibular placode from which placodal cells are detached *en masse*.

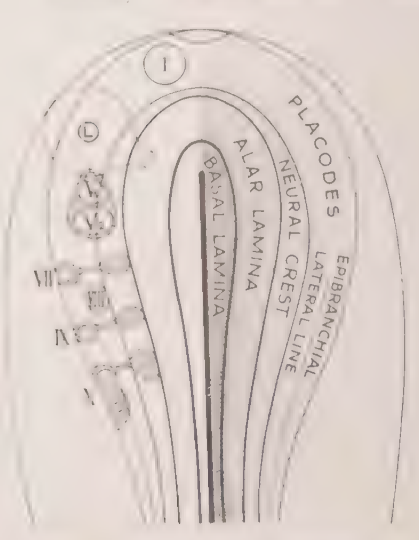

Fig. 11. Scheme of the neurogenetic ectodermic areas in a young reptilian neurula. I = olfactory placode, L = eye lens, V¹ = ophthalmicus component of V ganglion, composed from a larger placodal part and a lesser neural crest part. For the maxillo-mandibular component (V²) both parts are nearly equal The VII-, IX, and X ganglion are composed of cells from the neural crest area and from the epibranchial area. Eventua components from the rudimentary dorso-lateral placodes ar omitted. VIII = otic vesicle with otic ganglion, entirely derive from dorso-lateral (lateral line) placodal area. At the cranial en the hypophyseal placode. This figure chould be compared wit the similar one of a young Amphibian neurula (Fig. 14, 1941)

consists of an ectodermal thickening, which is now not invaginated any longer but which still sheds cells in a minor degree. The V^2 ganglion, which is connected by a broad radix, existing of many cells and fibres, with the medulla oblongata, still has a close contact with the epiderm from which it certainly still receives some few cells. Both ganglia contain some cells with very clear and large nuclei but in a much lesser quantity than the other branchial ganglia. It cannot yet be said whether these come from the placodes, from the neural crest or from both.

Also from the *Chelydra* and *Coluber* series it appears that the maxillo-mandibular part of the trigeminal ganglion after fusion with the ophthalmic component still has contact with the epiderm during a long time. The place of this contact lies a little caudally from the place of entrance of the radix of the ganglion in the medulla oblongata. The V^2 ganglion shows itself as a large, round mass of cells, which can occupy the whole space between the medulla oblongata and the epiderm. The trigeminal ganglion in Reptiles is extraordinary voluminous. Also the proliferation of the placodes and especially that of the V^2 placode, stretches itself over a considerable time. Only the trigeminal placodes in *Lacerta* do not seem to be so important for the formation of the trigeminal ganglion as those in the genera mentioned above. As we saw already, the V^1 placode was insignificant in this animal and the same can be said, although not to such a degree, for the V^2 placode. It is true that in one of our *Lacerta* series it is clearly visible how cells are pushed off in a cluster from a V^2 placode but the latter is, properly speaking, not thickened and proliferation is only confined to some few succeeding sections. But also here a close contact remains during a long time between the maxillo-mandibular part of the trigeminal ganglion and the epidermis.

Mesectoderm.

It is always very difficult to state exactly the formation of mesectoderm without experiments. This must especially be the case in our investigation where relatively little material was available. Yet there are some indications that also in the Reptiles, here investigated, ectoderm, other than that of the neural crest, contributes to the supporting tissue of the head. In our very young embryos of *Mabuia* in which a lens pit has just appeared and the hyomandibular pouch has just got into contact with the ectoderm, one can see how the ectoderm of the anterior part of the head proliferates in diverse places without forming a circumscript placode. The ectoderm here is, generally spoken, not much thickened any more, but localised masses of cells occur in it. On these places cells migrate in the subjacent mesenchyme and the continuity of the basal membrane is broken here. Such a situation is shown in Fig. 10 where a cell is seen, migrating from the dorso-lateral preotic ectoderm in a *Mabuia* embryo. Because these cells are not derived from circumscript placodes and migrate before the time at which there is a beginning of ganglion formation, it is only logical to assume that this more or less diffuse production of ectodermal cells has the task to increase the amount of mesenchyme, derived from other sources. It is known that such a thing can also happen in Ichthyopsida where especially the placodal areas are considered as suppliers of mesectoderm by some authors. Especially it is described by VEIT (1924, 1939) in *Lepidosteus* and *Petromyzon* where the whole lateral ectoderm of the head contributes in large quantities to the formation of the mesenchyme of the head. A similar situation is to be seen in young *Coluber* embryos of the same stage of development. Behind and somewhat above the eye there exists a region of thickened ectoderm which often consists of more than one layer of cells. In some places the membrana basalis has disappeared and cells migrate out of the ectodermal context.

We did not have the opportunity to observe the formation of mesectoderm from the placodes themselves and must confine us here to the demonstration of the great probability of mesectoderm formation out of the lateral ectoderm of the anterior part of the head in Reptiles.

Discussion.

Very definitely it appears that all cranial ganglia, the VIII excepted, in the genera of Reptiles under investigation, are almost exclusively formed by two components, one of the neural crest and one, rather considerable, of ectodermal placodes. The placodes which contribute to the branchial ganglia VII, IX and X are all situated epibranchially, in the dorsocaudal wall of the branchial clefts. The old view that the neural crest component of the ganglion would grow downward and that, after getting into contact with the epibranchial ectoderm, the placodal cells would migrate and mix themselves with the neural crest elements, does only hold good in a restricted sense, at least in Reptiles. It is the placodal cells which migrate over a shorter or longer distance dorsomesially towards the neural crest part of the respective ganglion, forming sometimes a real procession of cells as is especially to be seen in Fig. 6. The localised primitive epibranchial thickenings, which we call placodes from the moment when proliferation starts, are preceded by a general branchial thickening, present in the whole branchial area including also the area where in Ichthyopsida the dorso-lateral placodes are formed.

As a lateral line system is lacking in Reptiles, it is to be expected beforehand that the dorso(acustico)-lateral placodes which produce in Ichthyopsida the lateral line sense organs and their innervating ganglioncells, will be absent or only present in a rudimentary form. In two embryos such placodes were found, one situated just dorsally to the epibranchial VII placode in a *Chelydra* embryo, the other just dorsally to the epibranchial IX placode in a *Tropidonotus* embryo. The cells, migrating from these placodes are very similar in histological character to the epibranchial placodal cells. Thus we can wholly confirm the statement of EVANS (1934) who describes the presence of this kind of placodes connected with the VII and IX complex in some rare specimens of his Gecko embryos, in varying degrees of separation from the respective epibranchial placodes. In this connection one could think of an extension in a dorsal direction of the respective epibranchial placode, the more easily so because these rudimentary dorso-lateral thickenings, which occur only in a few succeeding sections, are so indistinctly separated from the epibranchial placodes which are lying just ventrally of them. Also the simultaneity of proliferation with the respective epibranchial placode points in this direction. It may be kept in mind as well, that the general branchial thickening from which the epibranchial placodes are developed also includes the dorso-lateral placodal area. That in a much younger stage a clear evidence is given by the dorso-lateral ectoderm of its capacity for proliferation is already mentioned in the chapter about the mesectoderm. Likewise in Amphibia the dorso-lateral placodes are not always clearly separated from the corresponding epibranchial ones but here the former begin earlier with proliferation. In our material a rudiment of a dorso-lateral X placode could not be found as happened in one case in the material of EVANS.

We think that those rudimentary acustico-lateral placodes are to be looked upon as reminiscences of phylogenetic older stages which still persevere, but that their cells do not reach a function, connected with a lateral line system. May be these placodes produce mesectoderm which is also the task of this kind of placodes in Ichthyopsida.

A special investigation concerning the genesis of the VIII ganglion was not carried out. More and more the view is held that the labyrinth is only a highly specialised lateral sense organ (1941). Because all lateral line sense organs, with the ganglioncells innervating them, are derived from the acustico-lateral placodes, it is to be expected beforehand, that also the octavus ganglion will arise from an acustico-lateral placode only, in casu the otic placode or the otic vesicle. For the latter fact convincing evidence is given by some authors concerning Amphibia and suggestions in this direction concerning Aves and Mammalia are made by others (1941). We also accepted this view for Reptilia and worked it up in Fig. 11, from which the genesis of the cranial ganglia is to be learned and the parts which are taken in it by the neural crest area and the placodal areas. The scheme represents the head of a young reptilian neurula stage in dorsal view and demonstrates how the ectodermal areas of prospective basal plate, alar lamina, neural crest and placodes succeed each other from medially to laterally. All these areas have primarily a neurogenetic potention, perhaps that the realisation of this potention is different according to the diverse Vertebrate classes. The neurogenetic potention of the basal plate and the alar lamina which constitute together the neural tube, is most completely realised in every Vertebrate class. Going laterally in the ectoderm, the realisation of this neurogenetic potention decreases. The neural crest cells have many functions which are not all specifically nervous. Thus for example the neural crest forms in Amphibia, besides ganglioncells, sympathetic and parasympathetic cells, pigmentcells and mesectoderm (ectodermal mesenchyme). The latter differentiates into connective tissue of the corium, pigmentcells, cartilage, leptomeninx and lemnoblasts (SCHWANNcells, also called peripheral glia).

The dorso-lateral (lateral line) placodal area gives origin to the lateral sense organs of which the labyrinth is a very specialised one, and to their innervating ganglioncells; from the epibranchial placodes ganglioncells are derived which, to the opinion of LANDACRE and many others after him, conduct gustatory stimuli. However, it is very probable that the epibranchial placodal cells have still another nervous function (see later). To all probability the placodal areas give also rise to mesectoderm besides to cells with the functions, mentioned before. The placodal areas and the neural crest area also contribute to the formation of the skin. Laterally from the placodal area the somatic ectoderm succeeds, which in normal circumstances is not fit to produce very specialised sense organs or nervous elements, although the potention for this is present as is shown in experimental circumstances (induction experiments of SPEMANN and his school).

Coming now to the Vertebrates under investigation, the Reptiles, it follows from the left side of Fig. 11 that all cranial ganglia are of a complex origin, the VIII ganglion excepted. The components of the V ganglion are composed of a placodal part which can be fairly large in Reptiles such as f.i. in *Seps* and in most other genera which were investigated, but which can be also only slight as was found in *Lacerta*, and a neural crest part. The VII, IX and X ganglia are composed of epibranchial placodal cells and of neural crest cells. The contribution from the rare rudimentary dorso-lateral placodes has been omitted. The fact that in Fig. 11 only two epibranchial placodes of the Xth ganglion are designed does not mean that there are only two branchial clefts connected with the vagus but that generally there are only two epibranchial placodes contributing to the vagus ganglion. EVANS found in some cases an epibranchial thickening above the third vagal cleft which he describes as an epibranchial placode. We also found this thickening in some rare cases. However, being not sure, that this third epibranchial thickening contributes as a rule cells to the vagus ganglion, thus forming a real placode, we did not draw it in our scheme. The VIII ganglion is figured as a product of the dorso-lateral placodal area, accepting the view, mentioned before, that this ganglion has its origin wholly in the otic vesicle. Comparing Fig. 11 with the corresponding scheme for Amphibia (Fig. 14, 1941) it is to be seen that in Reptiles the relative contribution of the maxillo-mandibular placode to the trigeminus ganglion is larger than in Amphibia. Further differences are the lack of contribution from the dorso-lateral placodal area to the VII, IX and X ganglia and the lack of primordia for the lateral line sense organs in this area. The more limited contribution of epibranchial placodes to the X ganglion in Reptiles is only seeming. In our scheme for Amphibia we certainly figured more epibranchial placodes for the vagus than are generally in function. Also here the most occurring number is two but it may be three.

Thus, the most conspicuous difference between the two schemes exists in a considerable limitation of the dorso-lateral placodal area to produce sense organs and neurons. That the neurogenetic activity in this area may not wholly have ceased in Reptiles, besides the production of the labyrinth with its ganglion, would appear from the few cases in which still dorso-lateral placodes with migrating cells were found. It must, however, be kept in mind that nothing is known as yet about the function of these cells. Perhaps they are indeed neuroblasts, perhaps they are mesectodermal cells, *e.g.* lemnoblasts, which is more probable because the Reptiles certainly do not have a functioning lateral line system. Rather interesting in this respect is the very young embryo of *Mabuia*, already mentioned before (Fig. 9). It clearly appears from these facts that also in Reptiles the dorso-lateral ectoderm still preserves more generally its proliferating tendency.

The difference in histological character between the epibranchial placodal cells and the neural crest cells which constitute the VII, IX and X ganglia is very apparent as pointed out before. This difference is also figured by SAINT–RÉMY and PRENANT and more or less by EVANS but is not mentioned by these authors. According to the nomenclature used by the former authors in their figures, the ganglion geniculatum VII, the ganglion petrosum IX and the ganglion nodosum X would be all formed exclusively from the cells with their large, clear nuclei and large amount of protoplasma, which, following our investigation, are derived from the epibranchial placodes. The function of the neurons of these parts of the branchial ganglia is known as practically exclusively viscero-sensory, *i.e.* they lead general- as well as special visceral sensibility. That would mean that the neuroblastic epibranchial placodal cells which constitute these parts of the ganglia, are special visceral as well as general visceral in nature. Now it is certainly not allowed to draw this conclusion without knowing more exactly the further history of the neural crest cells and the epibranchial placodal cells which constitute these ganglia. This is a matter of investigation worth doing. From what I have seen in older reptilian embryos certainly the big placodal cells get the upper hand in the VII ganglion which is practically exclusively viscero-sensory, and it seems to me that most of the neural crest cells are developed into lemnoblasts. This would be quite in accordance with the view of VAN CAMPENHOUT (1937) that in *Gallus* the ganglia are nearly entirely formed by placodal cells whereas the task of the neural crest would only consist in furnishing SCHWANNcells. From this it seems very probable indeed that the big placodal cells have more than simply a gustatory function (LANDACRE), also keeping in mind the distribution of the rami of this part of the ganglia as mentioned before. In this respect also the results of the experiments of YNTEMA (1937) and STONE (1922) on *Amblystoma* may be cited. According to these authors the epibranchial placodes also produce general visceral elements.

It is difficult to understand the meaning of the histological difference between the placodal neuroblasts and neurons and the cells of neural crest origin on one side and the difference between the migrated placodal cells and non-differentiated placodal cells, still lying in the ectodermal context, on the other. VAN CAMPENHOUT (1937) also described a histological difference between the placodal cells and those of the neural crest in *Sus*. The former are more round, the latter are stellate. Also in some Fishes there is a difference between these cells (LANDACRE (1910) in *Ameiurus*; REED (1916) in *Acanthias*). In these latter cases, however, the placodal cells are stained darker and are smaller than the neural crest cells. Here an explanation has been given by the assumption that the placodal cells are differentiated into ganglion cells later than the neural crest cells. In older stages this histological difference is lost in contrast to the histological properties of the epibranchial placodal cells in Reptiles which remain, also after their differentiation into ganglion cells. The latter fact makes it easy to make a distinction, also in older stages, which part of the cranial ganglia is derived from the placodes and which from the neural crest. It is very remarkable that also some placodal trigeminal neuroblasts have the same histological character as the epibranchial placodal cells. This fact obscures still more an understanding of a possible parallelism between histological properties of cells and their function, all the trigeminal neurons practically having, so far as is known, another function than the epibranchial placodal cells.

Zusammenfassung.

1. Embryonale Serien von *Chelydra, Hemidactylus, Mabuia, Seps, Lacerta, Coluber* und *Tropidonotus* wurden untersucht auf das Vorkommen und die Genese von Epibran-

chialplakoden, dorso-lateralen Plakoden, Trigeminus Plakoden und Mesektoderm.

2. Die Epibranchialplakoden entwickeln sich aus einer allgemeinen, branchialen Ektodermverdickung. Später bleiben dann nur lokalisierte Verdickungen an der dorsocaudalen Wand der Kiementaschen übrig, die wir Plakoden nennen sobald sie zu proliferieren anfangen.

3. Epibranchialplakoden befinden sich immer an der dorso-caudalen Wand der VII, der IX und der beiden ersten X Kiementaschen. Die abgestossenen Zellen migrieren mehr oder weniger weit in dorso-mesialer Richtung um sich bei den Zellen des gleichen Komplexes zu fügen, die aus der Neuralleiste entstehen.

4. Die plakodalen Zellen zeichnen sich durch verschiedene charakteristische Unterschiede gegenüber den Elementen aus der Neuralleiste aus. Diese Unterschiede formen lange eine Andeutung für die Herkunft der Zellen in den Ganglienanlagen, auch noch nachdem die Neuroblasten angefangen haben sich zu Ganglienzellen zu differenzieren.

5. Dorso-laterale Plakoden wurden nur in zwei Fällen aufgefunden. In einem direkt dorsal von einer proliferierenden epibranchialen VII Plakode, in dem anderen in der gleichen Lage über einer gleichfalls proliferierenden epibranchialen IX Plakode. Es wird an die Möglichkeit gedacht, dass die austretenden Zellen Mesektoderm (ev. Lemnoblasten) liefern oder dass die Plakoden nur eine Ausbreitung des epibranchialen plakodalen Feldes sind, wofür es auch Argumente gibt.

6. Es werden ophthalmicus- und maxillo-mandibuläre Plakoden beschrieben, die bei den meisten untersuchten Reptilien einen sehr wesentlichen Anteil zum Aufbau des Trigeminusganglion liefern. Nur bei *Lacerta* scheint der plakodale Komponent geringfügig zu sein. Die Ophthalmicusplakode kann sich wie eine ektodermale Grube dartun.

7. Die Ausbildung von Mesektoderm aus dem Ektoderm des vorderen Teiles des Kopfes wird beschrieben.

8. Der Aufbau der kranialen Ganglien wird schliesslich an einem Schema erläutert und gezeigt wie alle, das Acusticusganglion ausgenommen, aus einem Neuralleistanteil und einem plakodalen Anteil hervorgehen. Es zeigt sich wie, im Vergleich mit dem Zustand bei den Amphibien, das dorso-laterale Plakodenareal seine Tätigkeit sehr beschränkt hat und nicht mehr wesentlich teilnimmt an der Zusammenstellung der branchialen Ganglien, obwohl Proliferation in diesem Areal noch wohl statt finden kann. Bei den Reptilien ist der Anteil der maxillo-mandibuläre Plakode am Aufbau des Ganglion GASSERI durchweg grösser wie bei den Amphibien.

9. Es wird näher eingegangen auf die Funktion der epibranchialen plakodalen Zellen. Andeutungen sind vorhanden, dass sie nicht nur Ganglienzellen für die spezifische viszerale Sensibilität (den Geschmack) liefern, sondern auch solche für die allgemeine viszerale Sensibilität.

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